

(Still More) Rule-Based Modeling of Biological Signaling

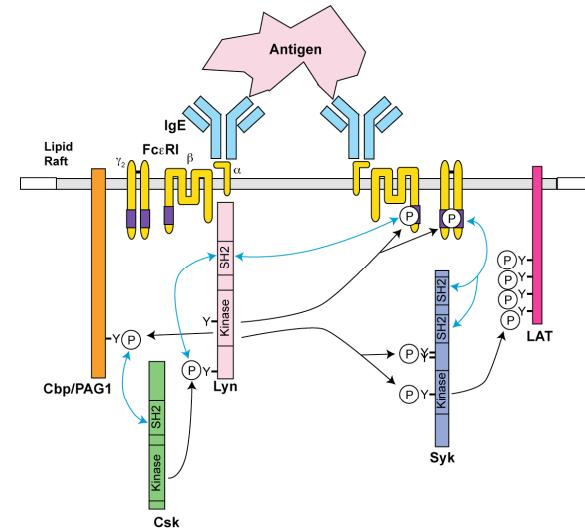
Simulation Methods, Examples, misc.

Jim Faeder

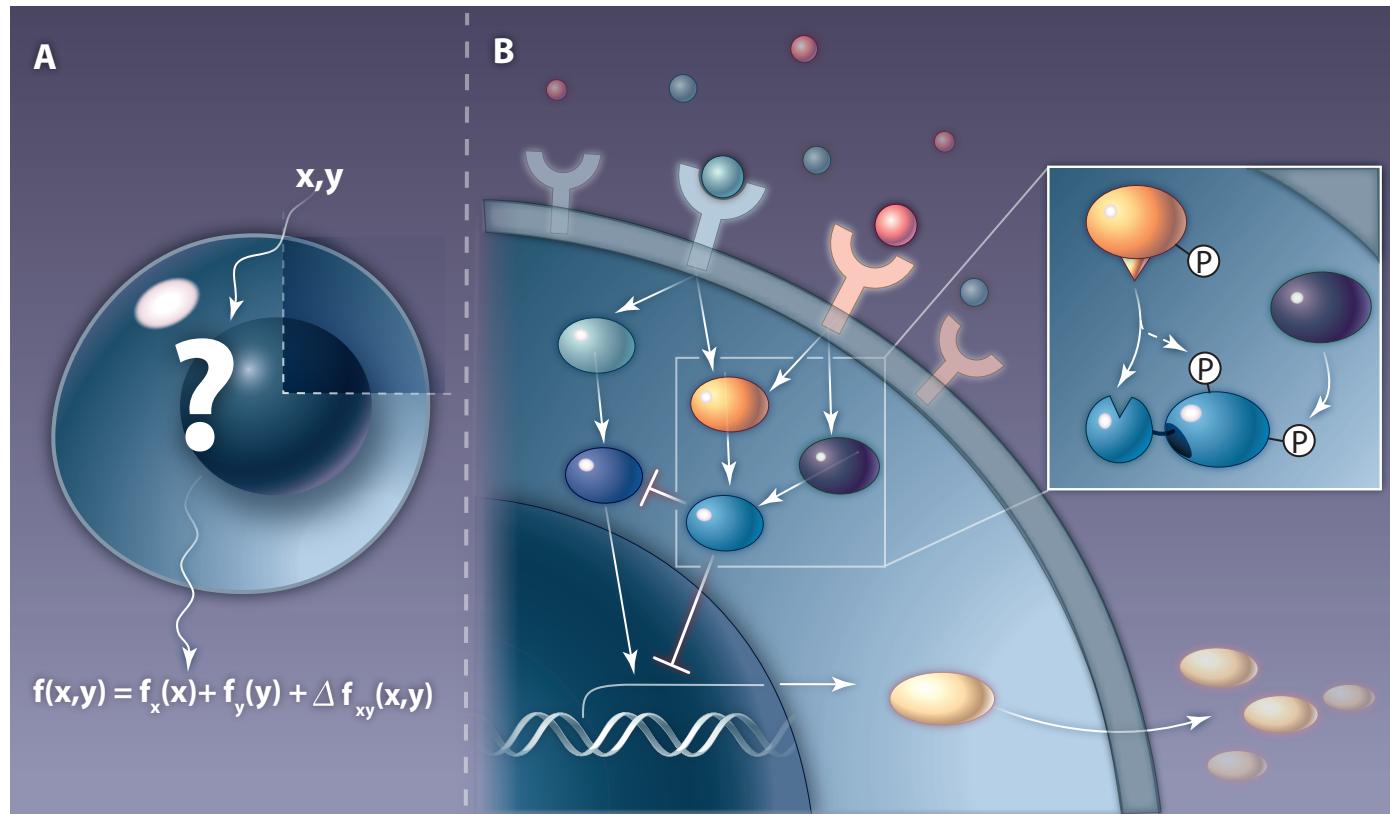
*Department of Computational Biology
University of Pittsburgh School of Medicine*

3rd q-bio Summer School

*Los Alamos, New Mexico
August 3, 2009*



Layers of Complexity in Cell Signaling

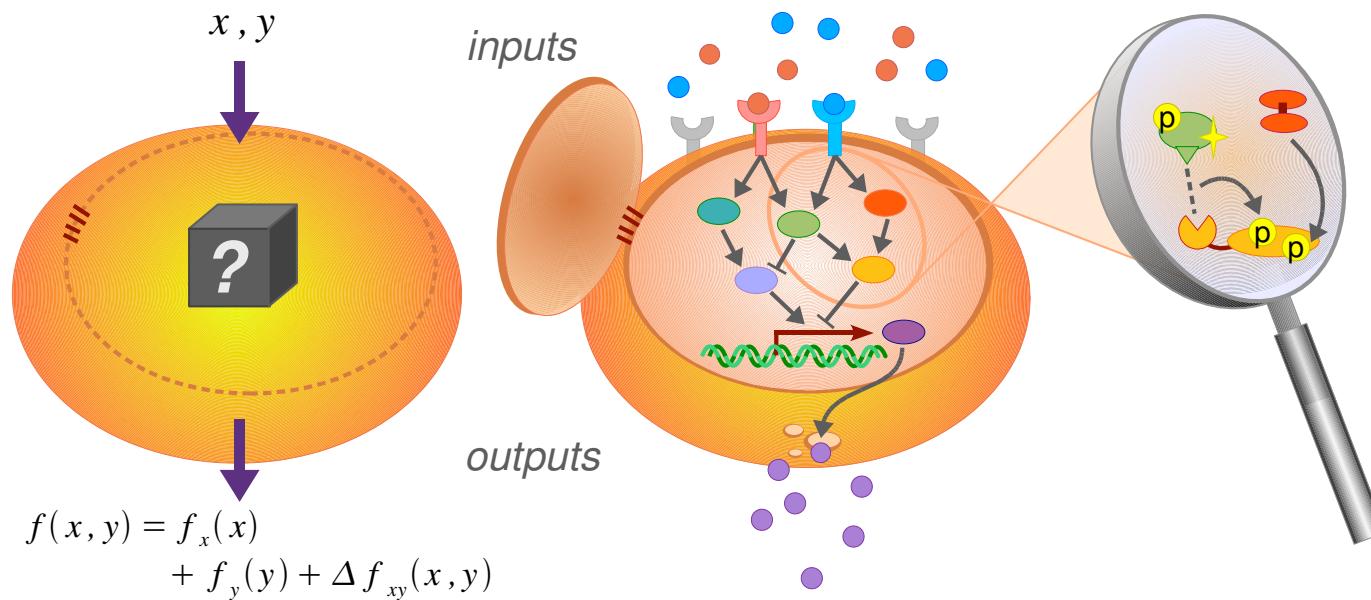


How does a cell combine multiple inputs?

How complex is the machinery of signaling?

How can we make sense of the complexity?

Layers of Complexity in Cell Signaling



Justin Hogg

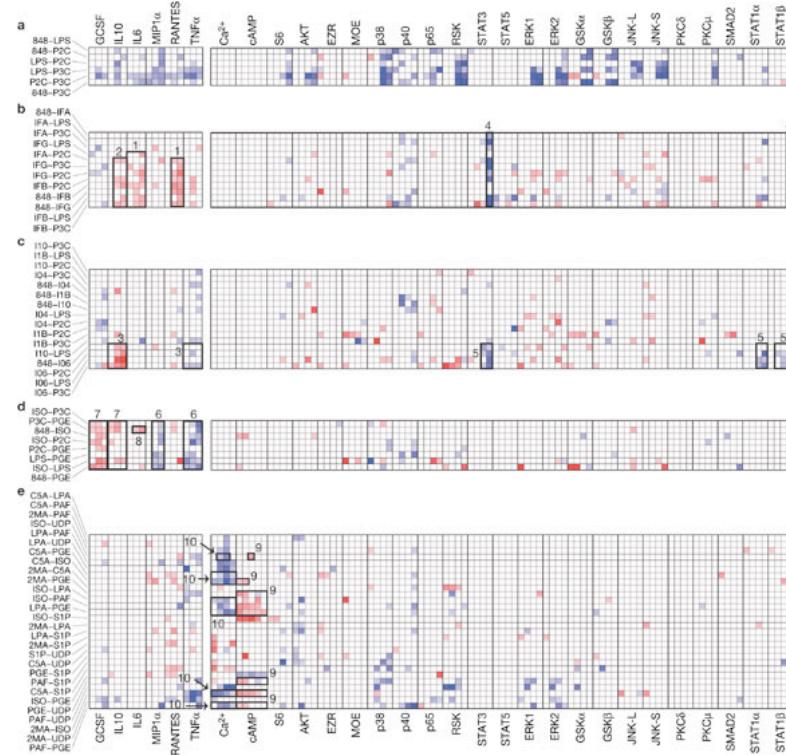
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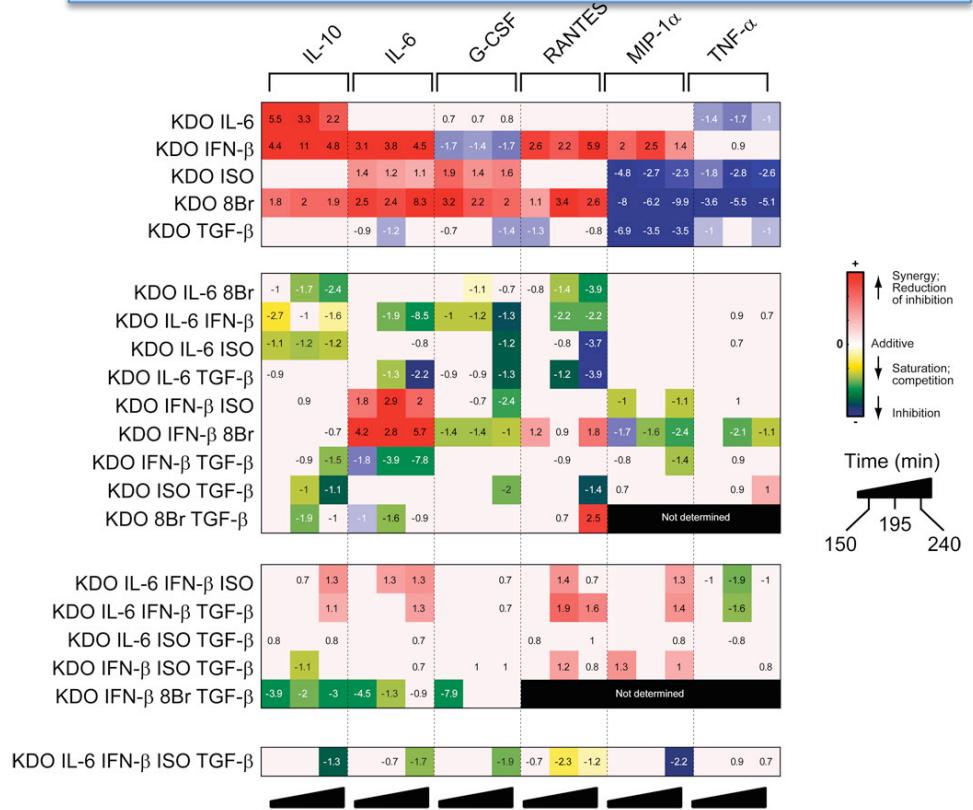
The Black Box Perspective

22 ligands / 231 combinations / 39 outputs



Natarajan et al. *Nat. Cell Biol.* 8, 571 (2006).

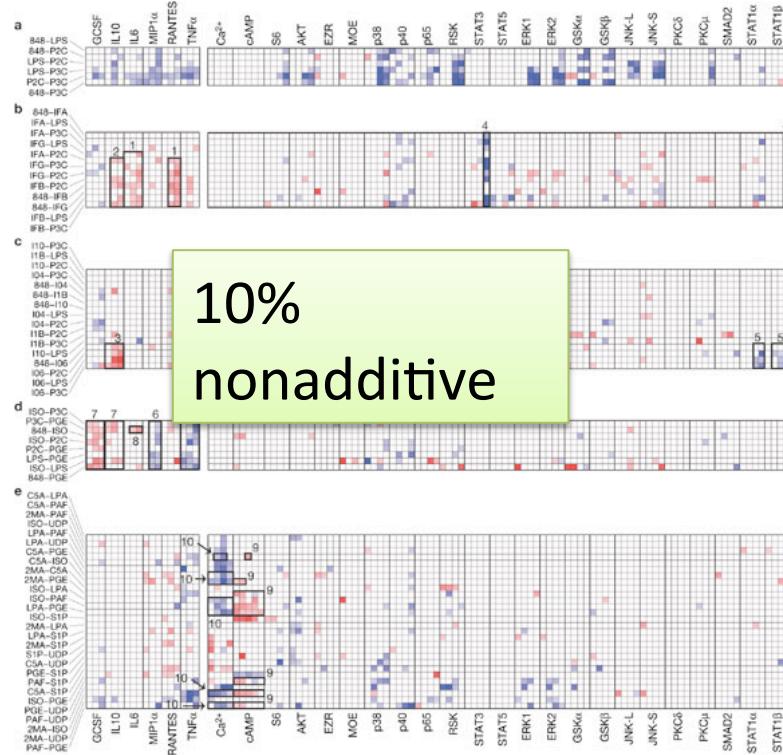
Responses to selected 3 way and higher combinations



Hsueh et al. *Sci. Signal.* 2, ra22 (2009).

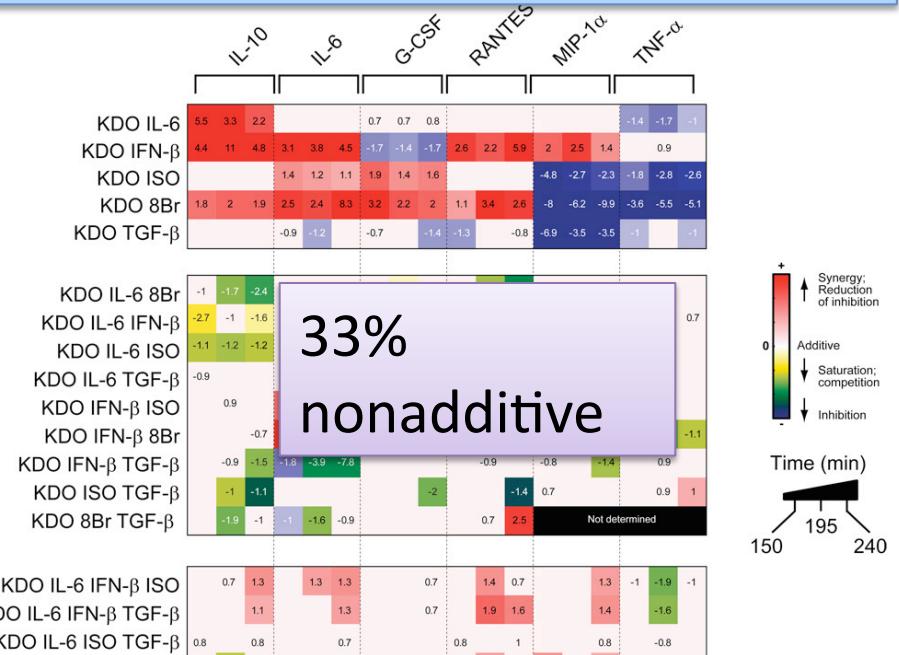
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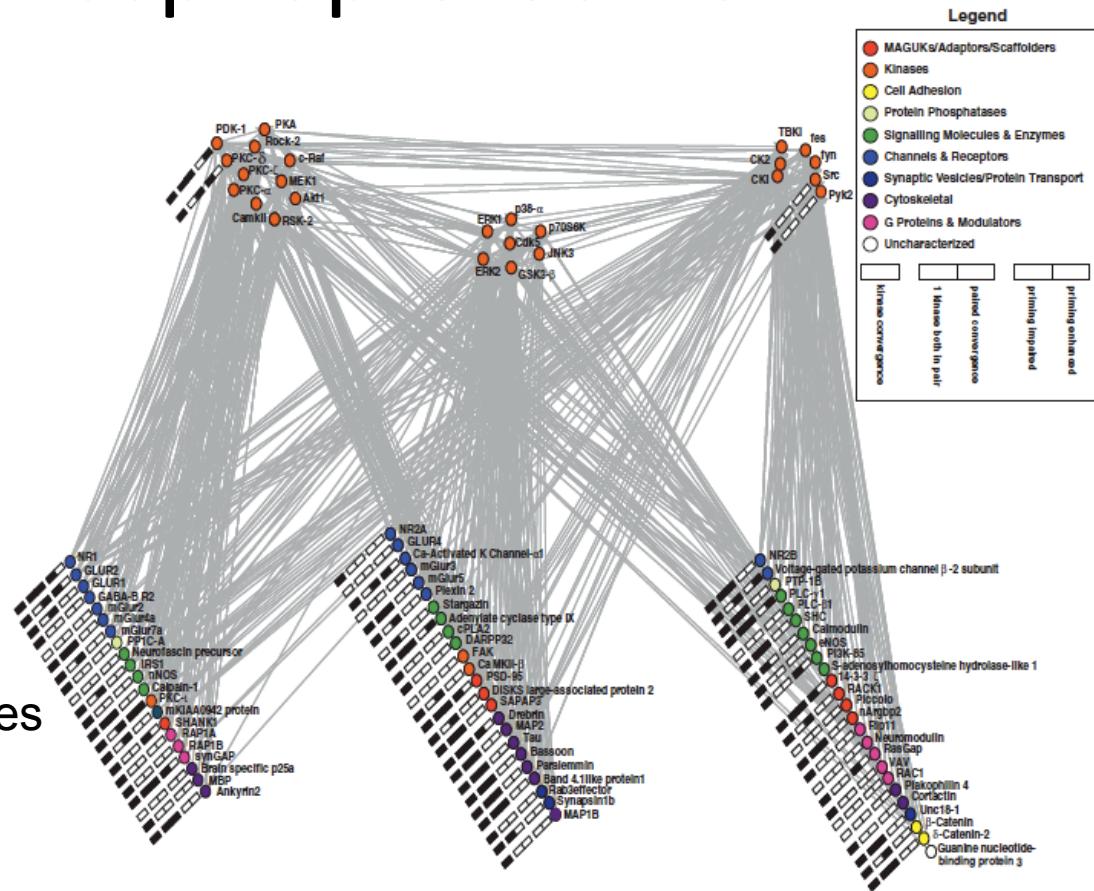


Hsueh et al. *Sci. Signal.* 2, ra22 (2009).

A Wiring Diagram The Phosphoproteome

Following stimulation by a single ligand (NMDA) of a single receptor (NMDAR)

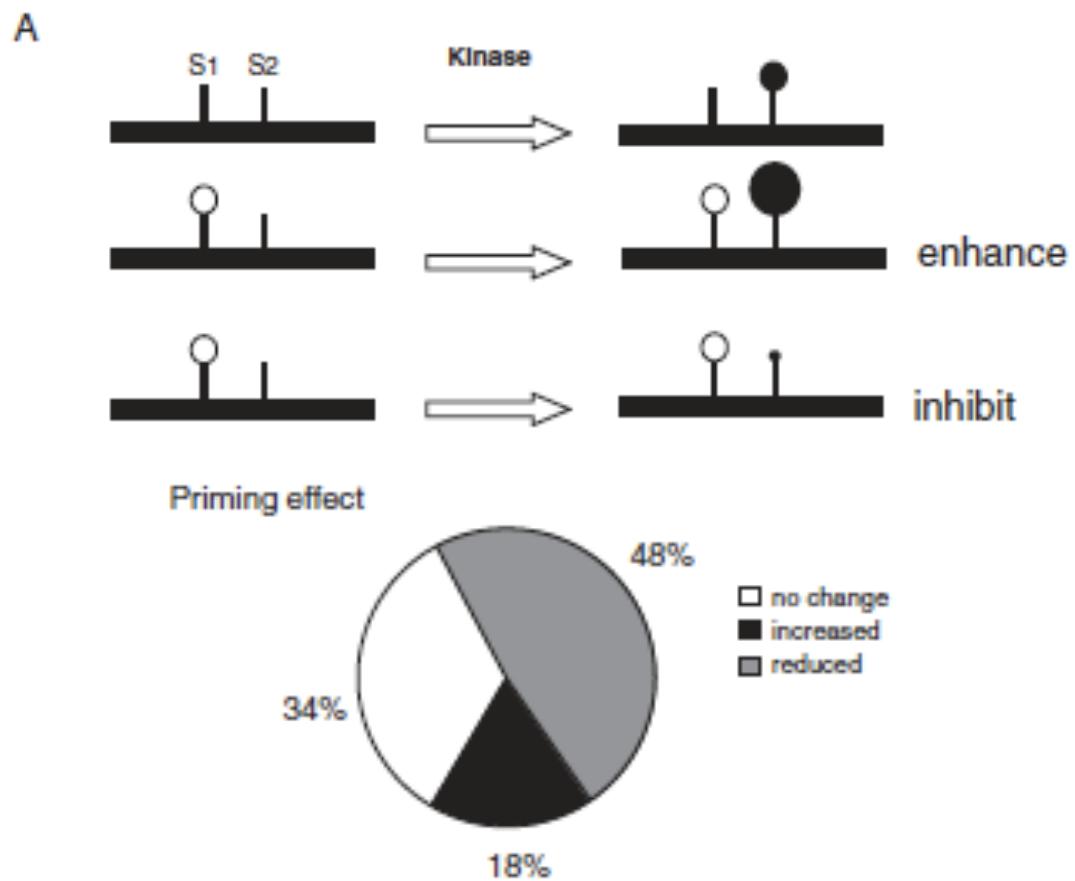
- 25 Kinases
- 300 sites / 166 observed
- 29 sites / kinase
- > 2 kinases / site
- 65% of proteins contain P-sites in binding domains



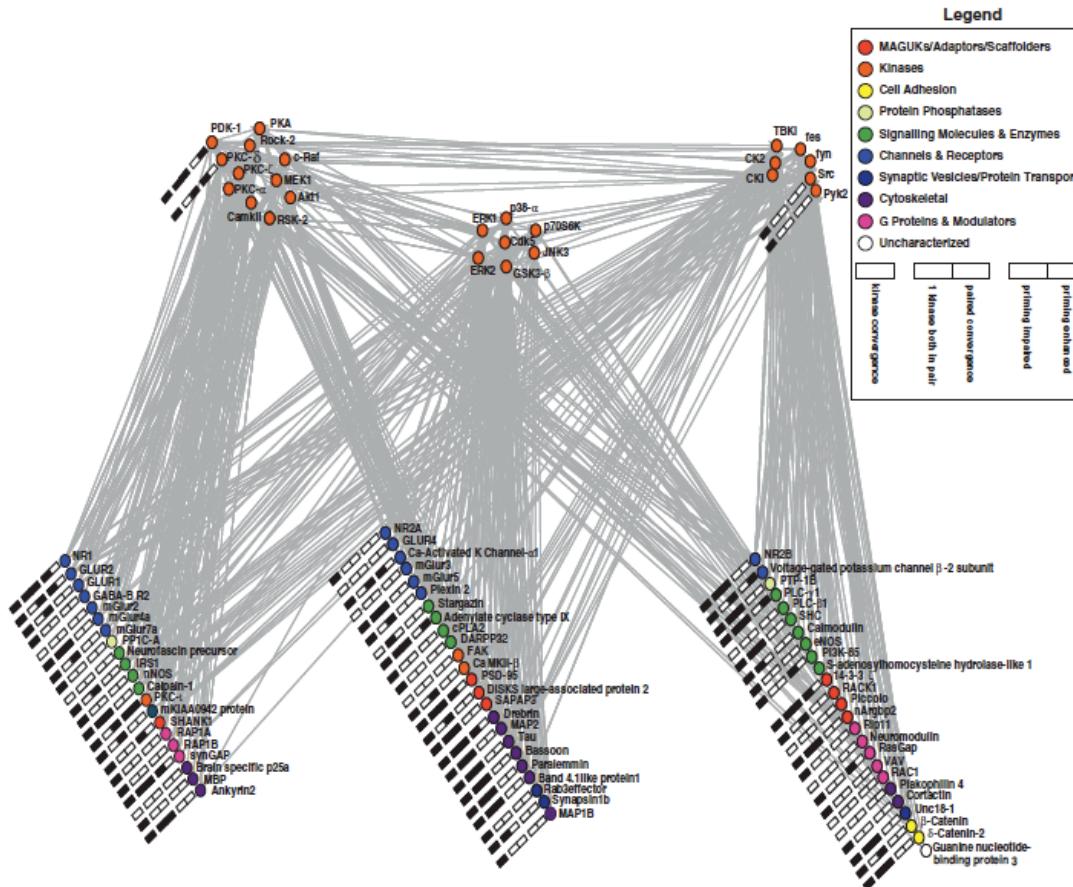
M. P. Coba et al. Neurotransmitters drive combinatorial multistate postsynaptic density networks. *Sci. Signal.* **2**, ra19 (2009).

Cooperativity in Phosphorylation

- 66% of 200 peptides exhibit “priming”

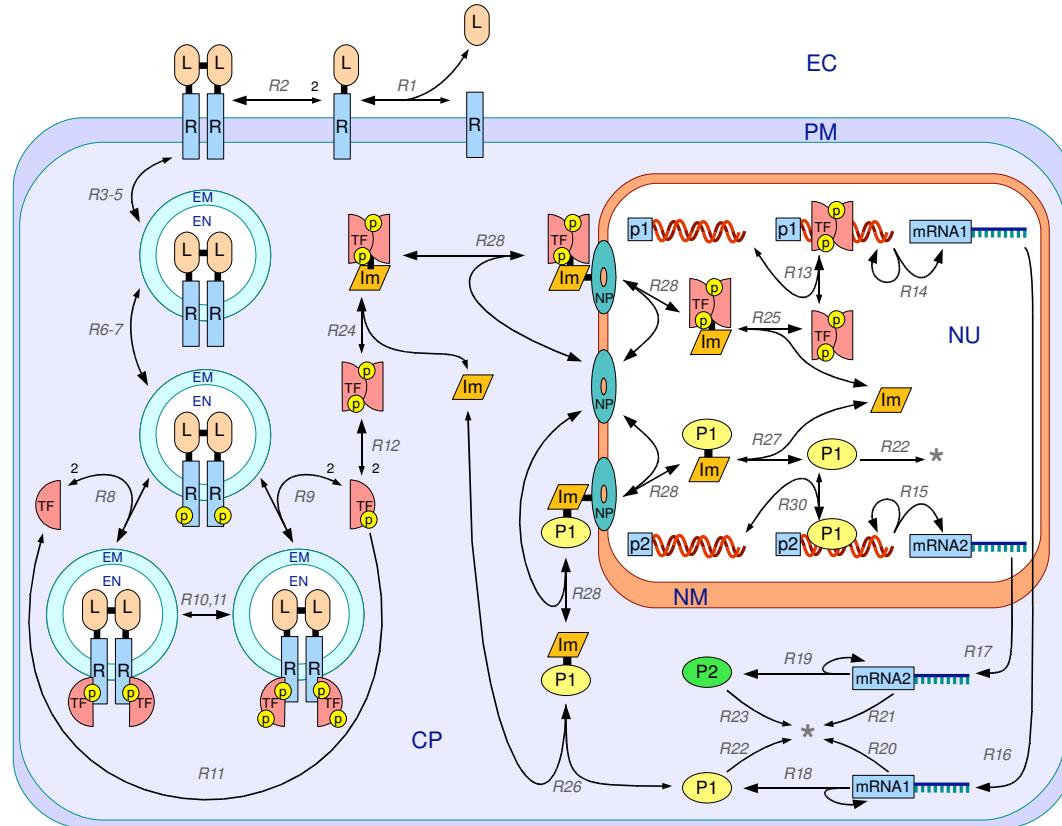


M. P. Coba et al. Neurotransmitters drive combinatorial multistate postsynaptic density networks. *Sci. Signal.* **2**, ra19 (2009).



W. S. Hlavacek, J. R. Faeder, The complexity of cell signaling and the need for a new mechanics. *Sci. Signal.* **2**, pe46 (2009).

Toward whole cell modeling

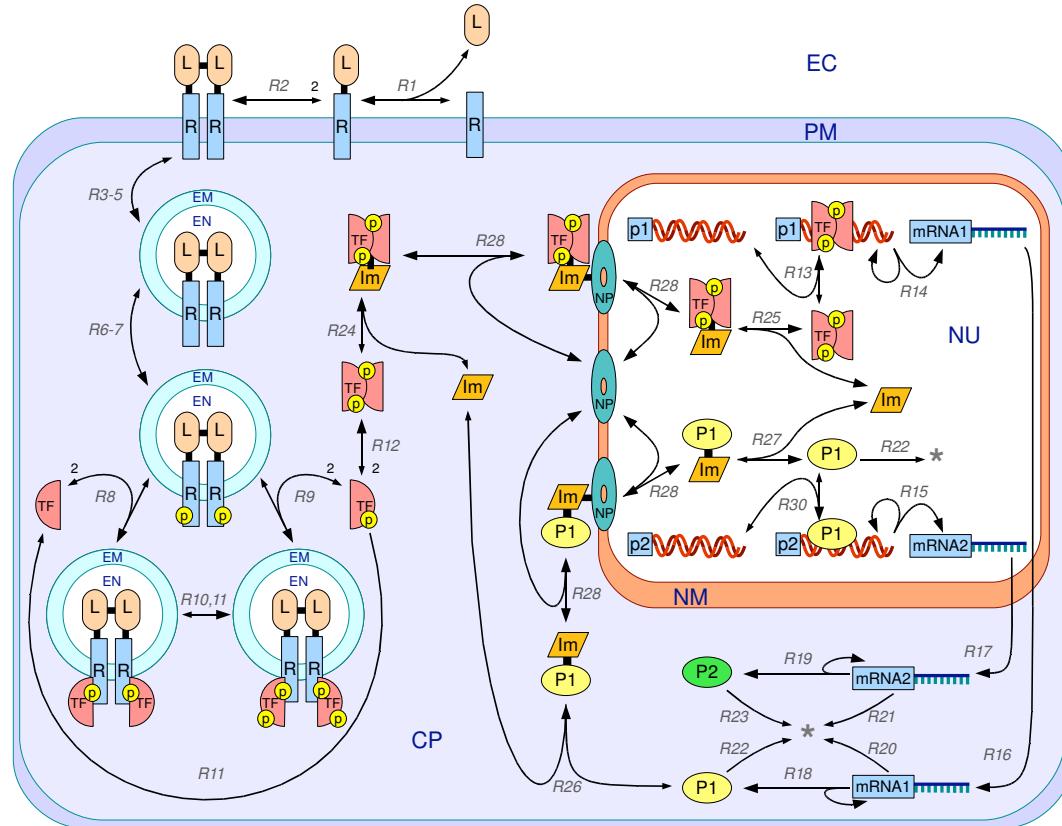


Justin Hogg

Key Questions:

- What parts of cell machinery are critical to the outcome?
- What controls the cell-to-cell variability?
- Can that variability be utilized by the organism to promote survival?

Toward whole cell modeling

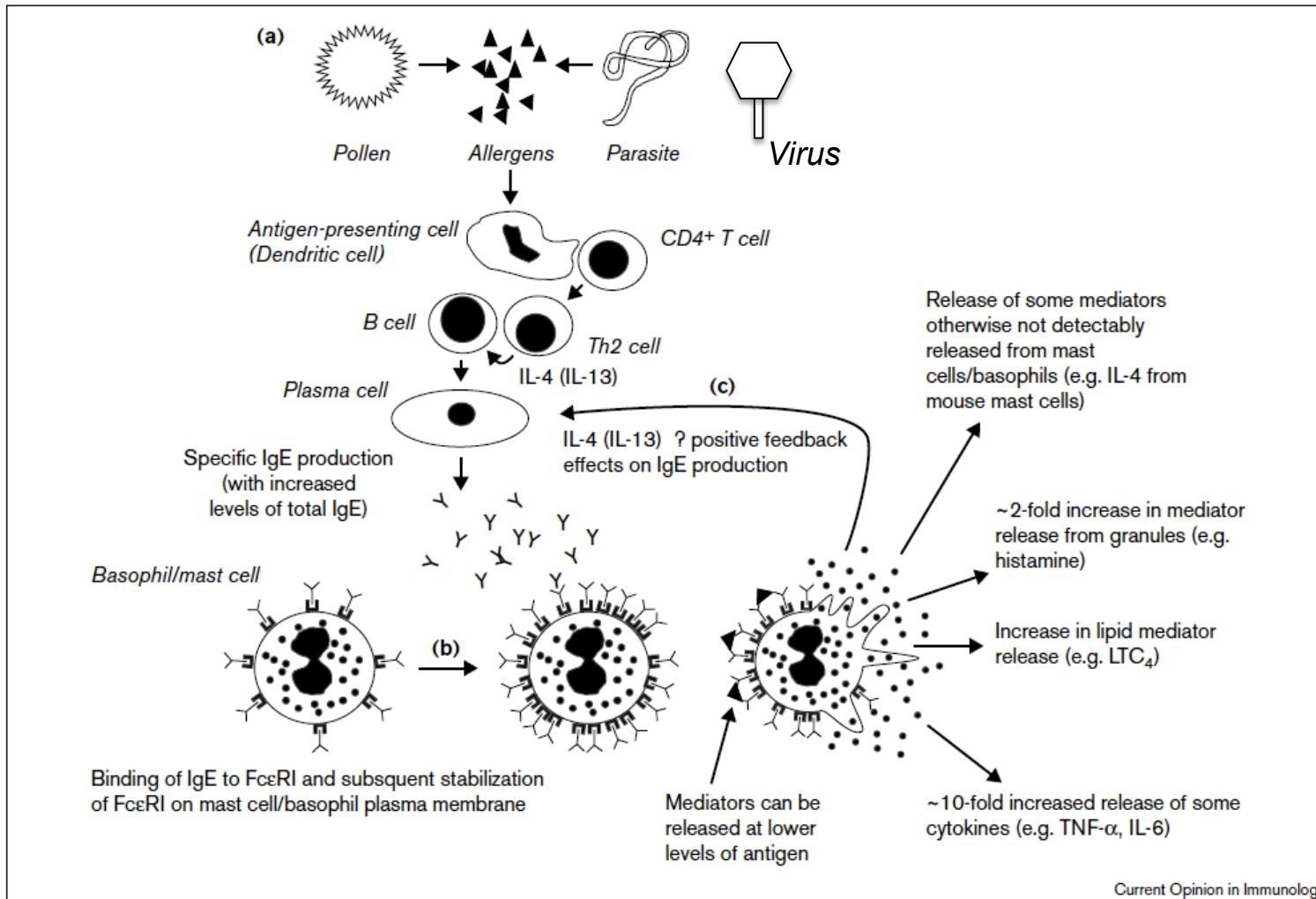


Justin Hogg

Approach:

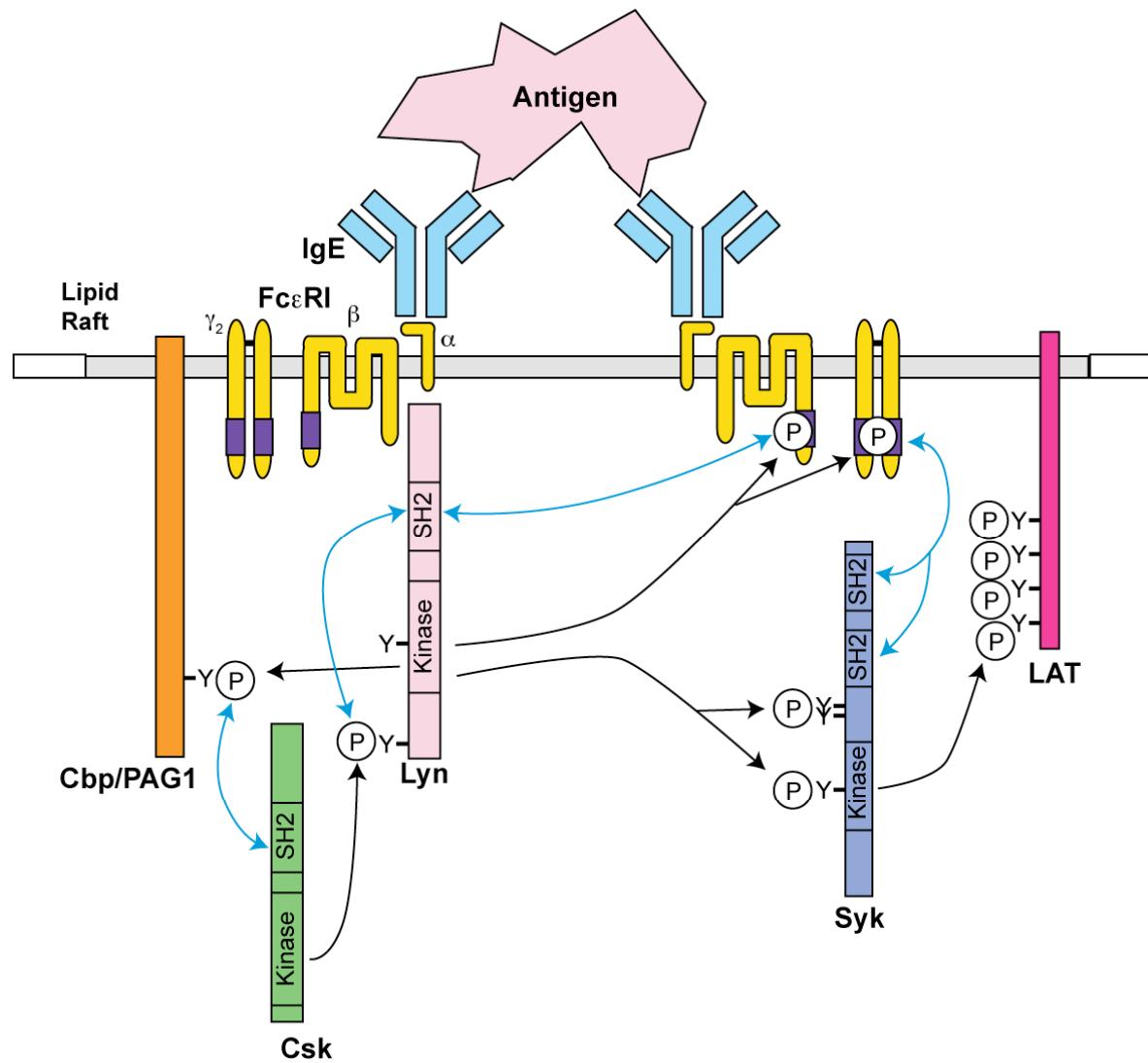
- Rule-based modeling
 - Phosphorylation and site-specific interactions
- Compartmental topology of the cell: *implicit* → *explicit*

Mast Cells / IgE Signaling in Host Defense

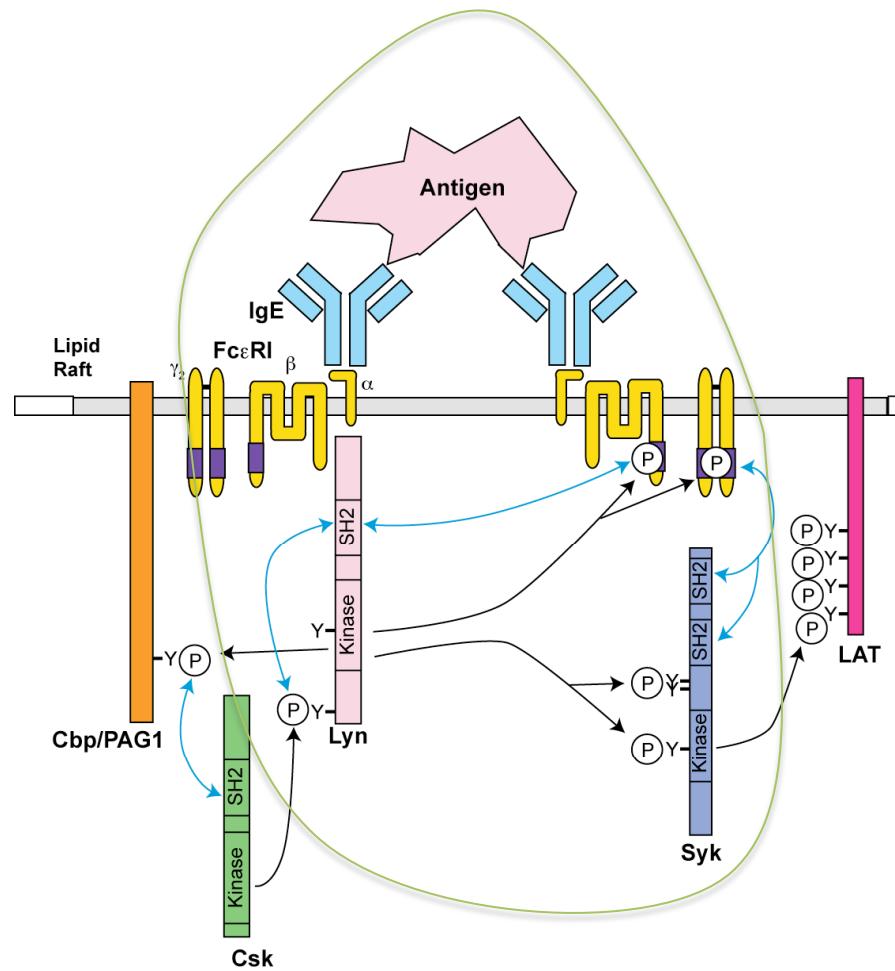


Wedemeyer et al., 2000

Early events in Fc ϵ RI signaling



Syk activation model



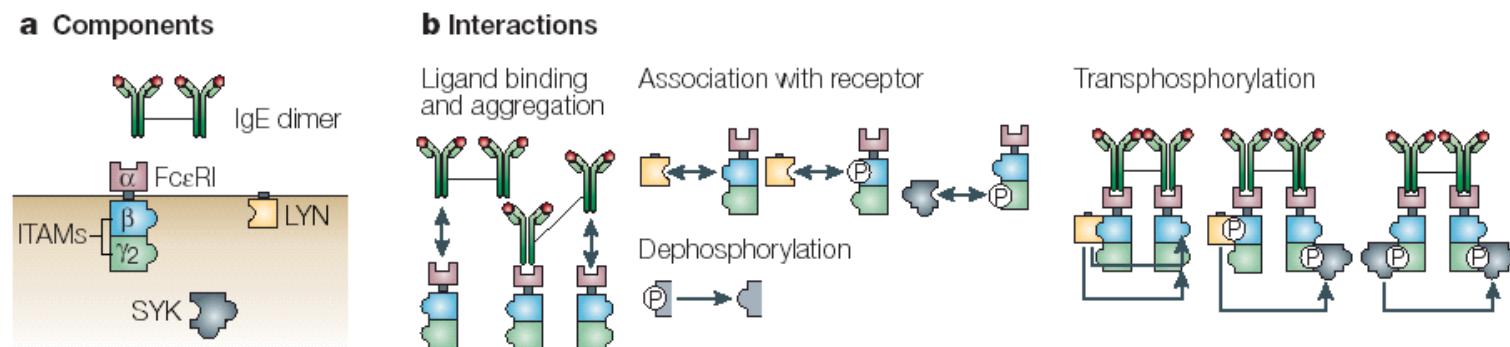
Key variables

- ligand properties
- protein expression levels
- multiple Lyn-FcεRI interactions
- transphosphorylation

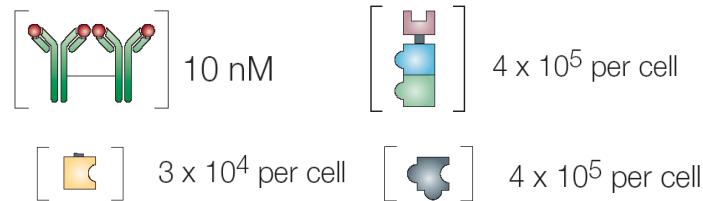
Mol. Immunol., 2002
J. Immunol., 2003

Standard modeling protocol

1. Identify components and interactions.



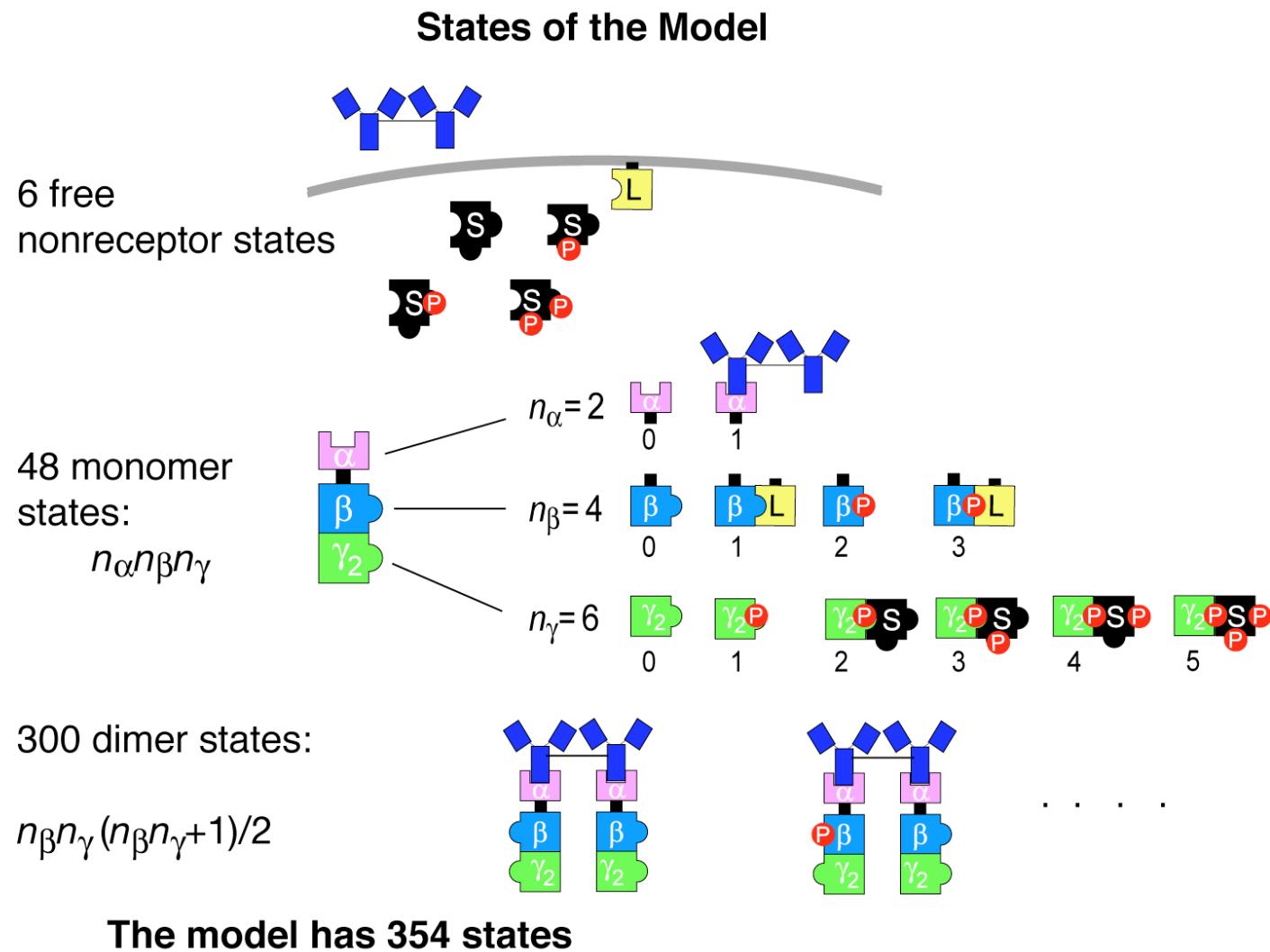
2. Determine concentrations and rate constants



3. Write and solve model equations.

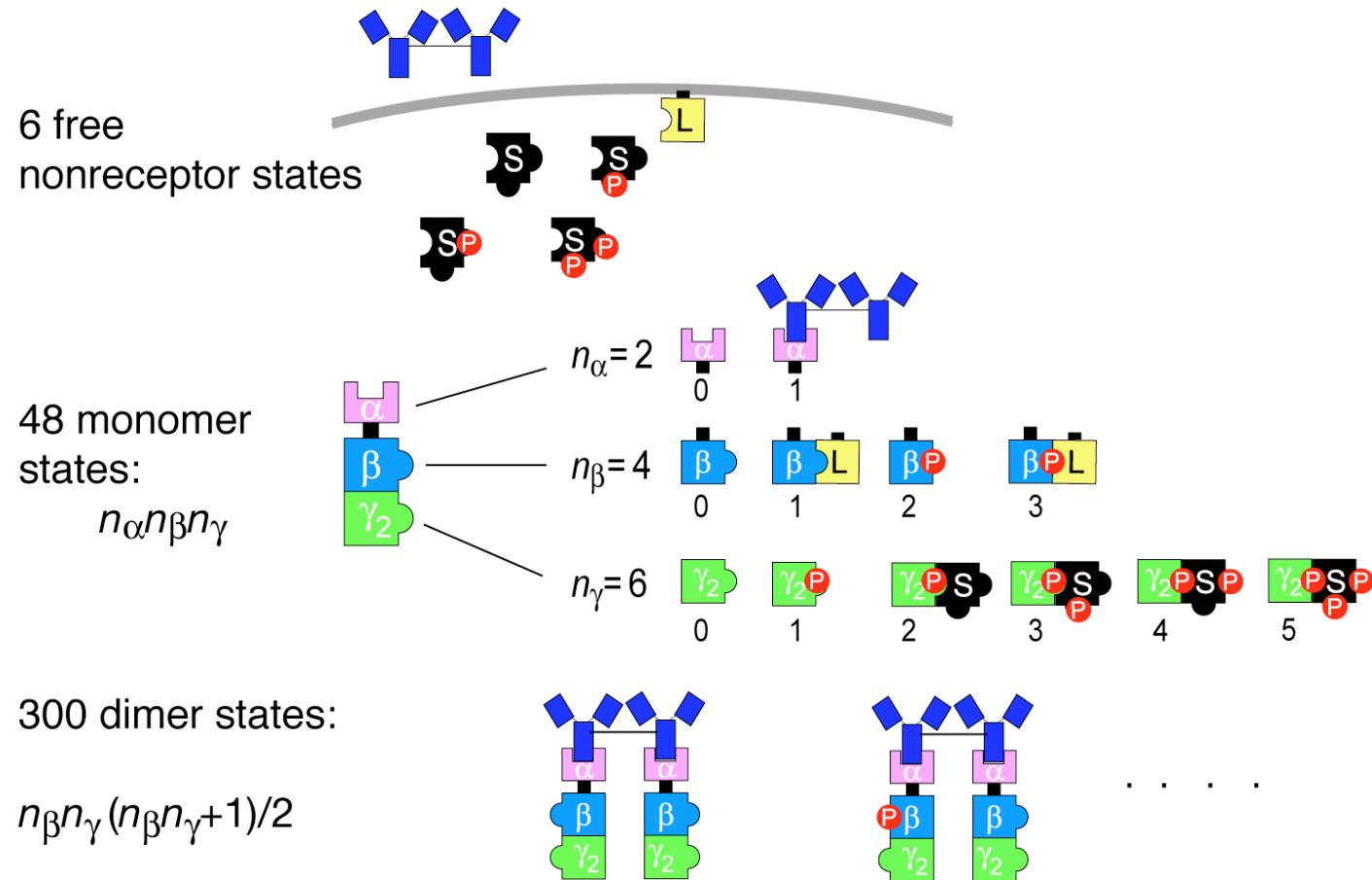
$$\dot{\mathbf{x}} = \mathbf{S} \cdot \mathbf{v}(\mathbf{x})$$

Combinatorial complexity



Combinatorial complexity

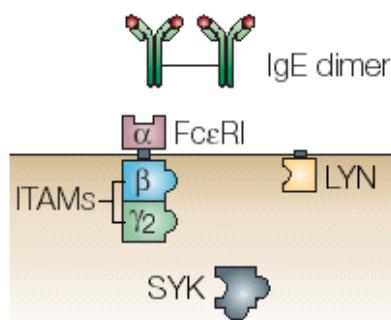
States of the Model



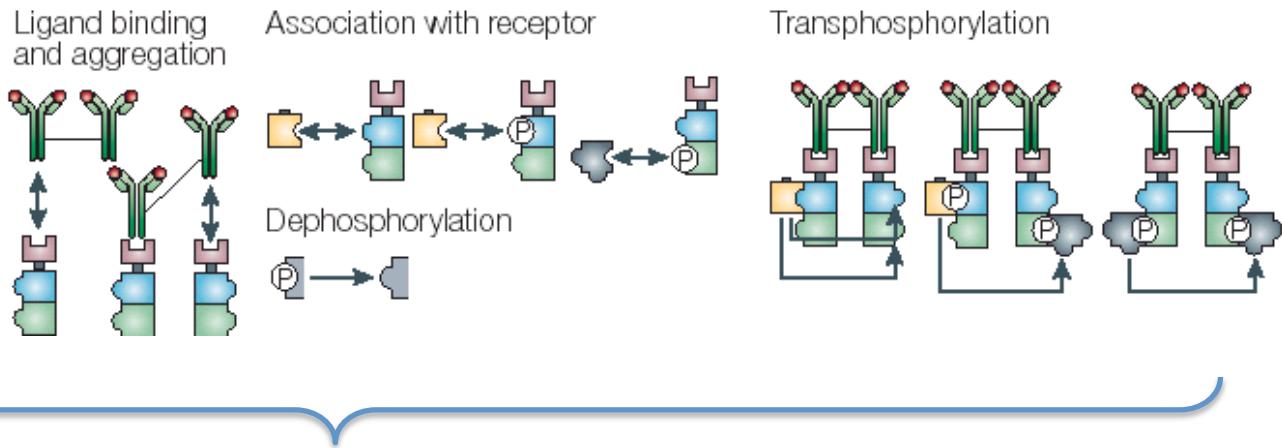
The model has 354 states (2954 if the ligand was a trimer)

Addressing combinatorial complexity

a Components



b Interactions

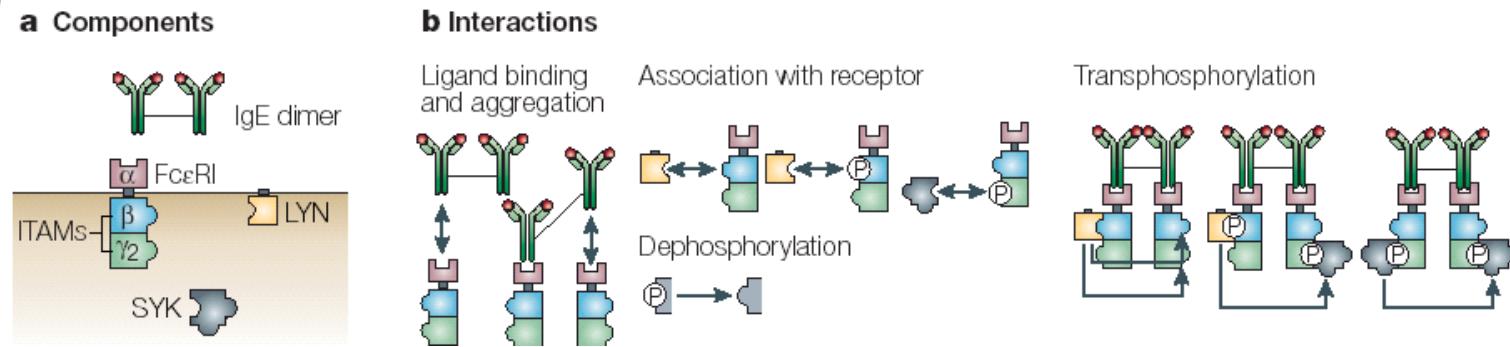


354 species / 3680 reactions

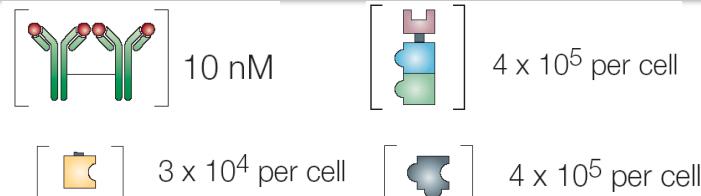
- Standard approach – writing equations by hand – won't work!
- New approach
 - Write model by describing interactions.
 - Automatically generate the equations.

Rule-based modeling protocol

1. Define components as *structured objects* and interactions as *rules*.



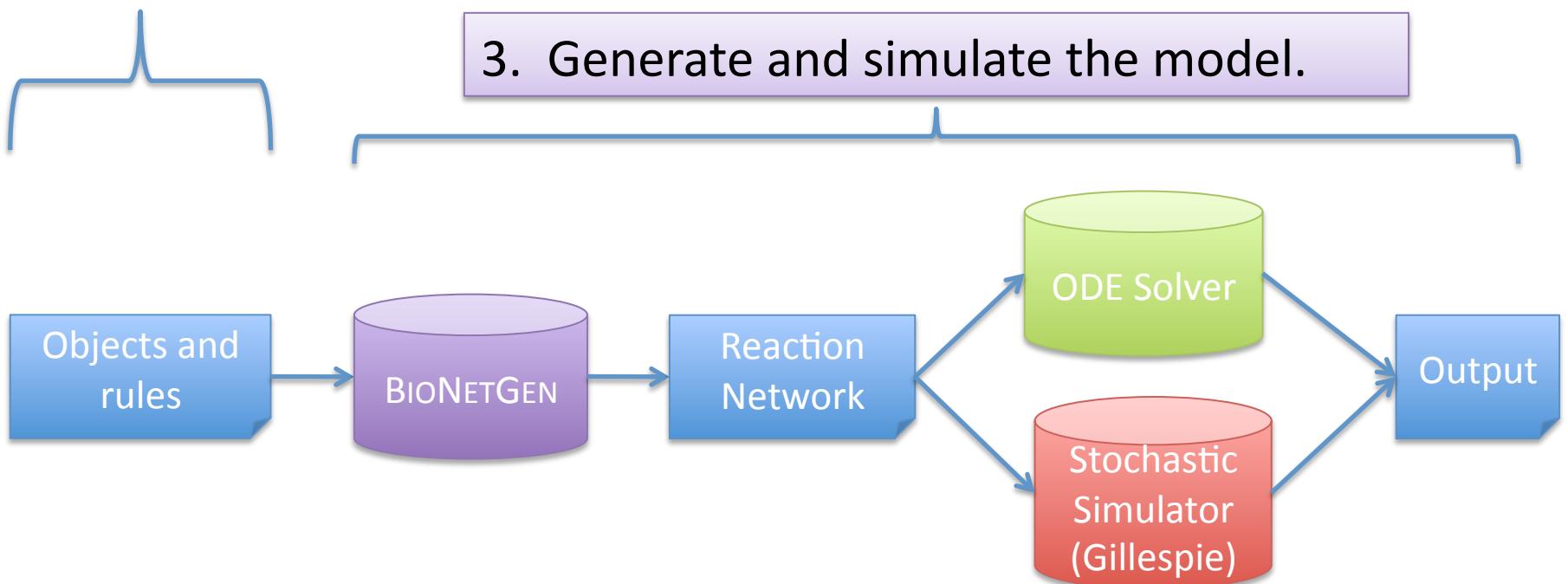
2. Determine concentrations and rate constants



3. Generate and simulate the model.

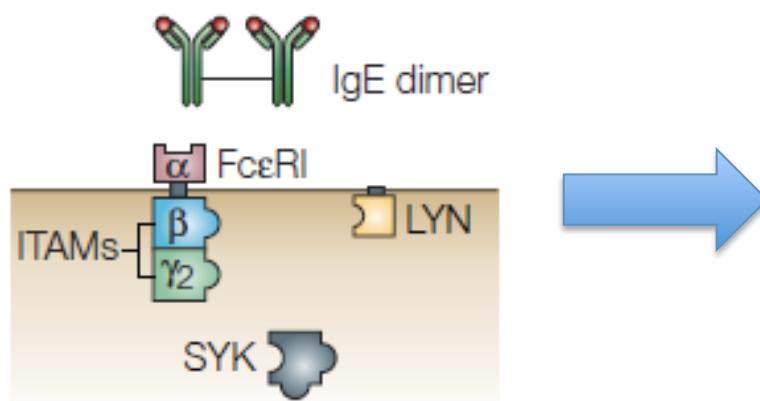
Rule-based modeling protocol

1. Define components as *structured objects* and interactions as *rules*.
2. Determine **concentrations** and **rate constants**



Defining Molecules

a Components

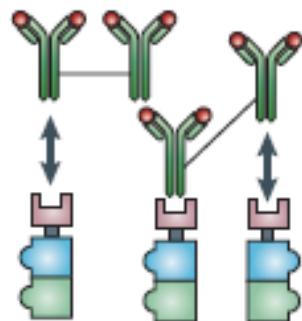


BioNETGEN Language

IgE(a,a)
Fc ϵ RI(a,b~U~P,g2~U~P)
Lyn(U,SH2)
Syk(tSH2,1Y~U~P,aY~U~P)

Defining Interaction Rules

Ligand binding
and aggregation



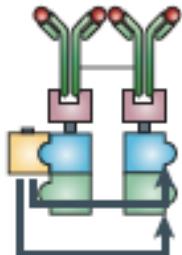
BioNETGEN Language

$\text{IgE}(a,\underline{a}) + \text{Fc}\epsilon\text{RI}(\underline{a}) \rightleftharpoons \text{IgE}(a,\underline{a!1}) \cdot \text{Fc}\epsilon\text{RI}(\underline{a!1})$

...

binding and dissociation

Transphosphorylation

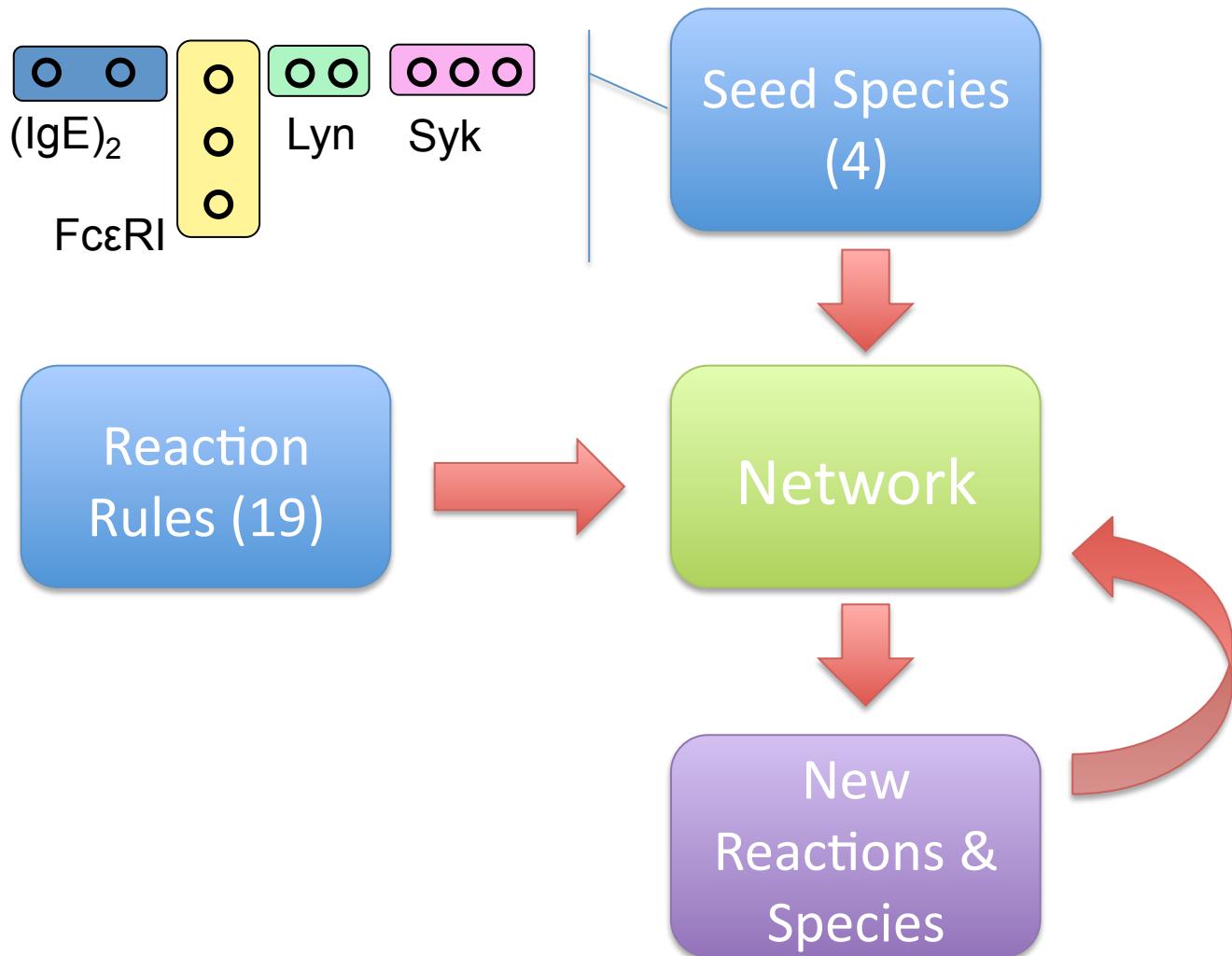


$\text{Lyn}(U!1) \cdot \text{Fc}\epsilon\text{RI}(b!1) \cdot \text{Fc}\epsilon\text{RI}(b\sim\underline{U}) \rightarrow \backslash$
 $\text{Lyn}(U!1) \cdot \text{Fc}\epsilon\text{RI}(b!1) \cdot \text{Fc}\epsilon\text{RI}(b\sim\underline{P})$

component state change

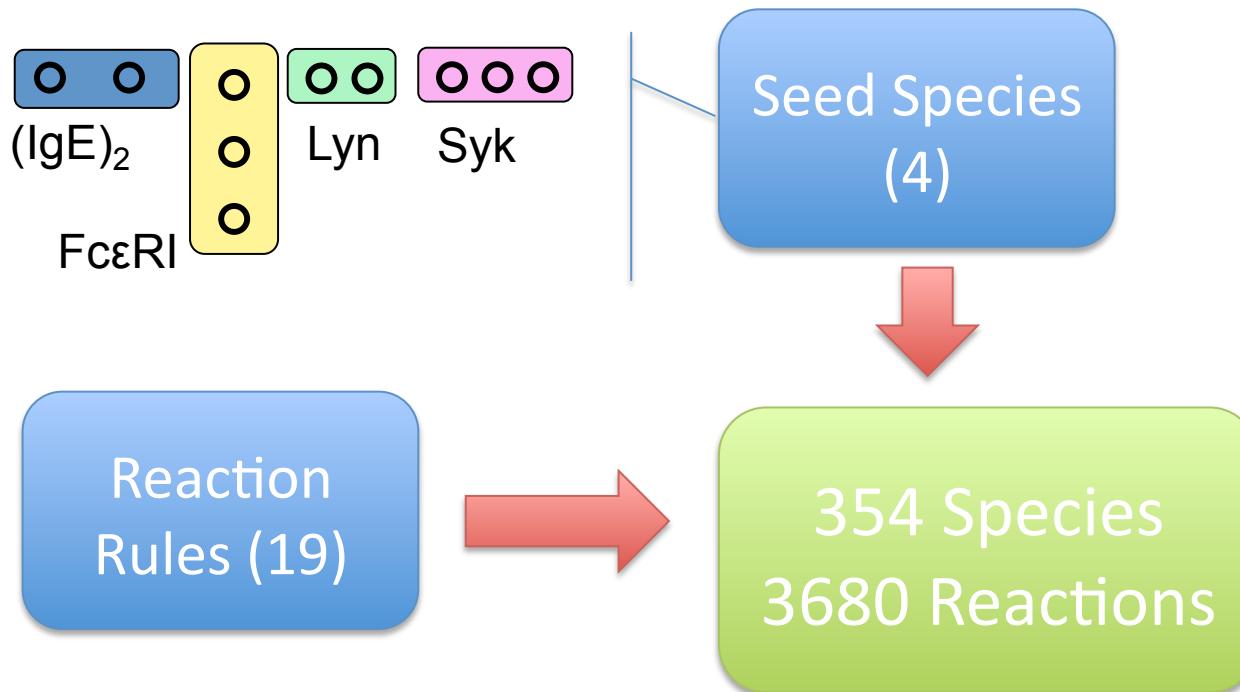
Automatic Network Generation

Fc ϵ RI Model



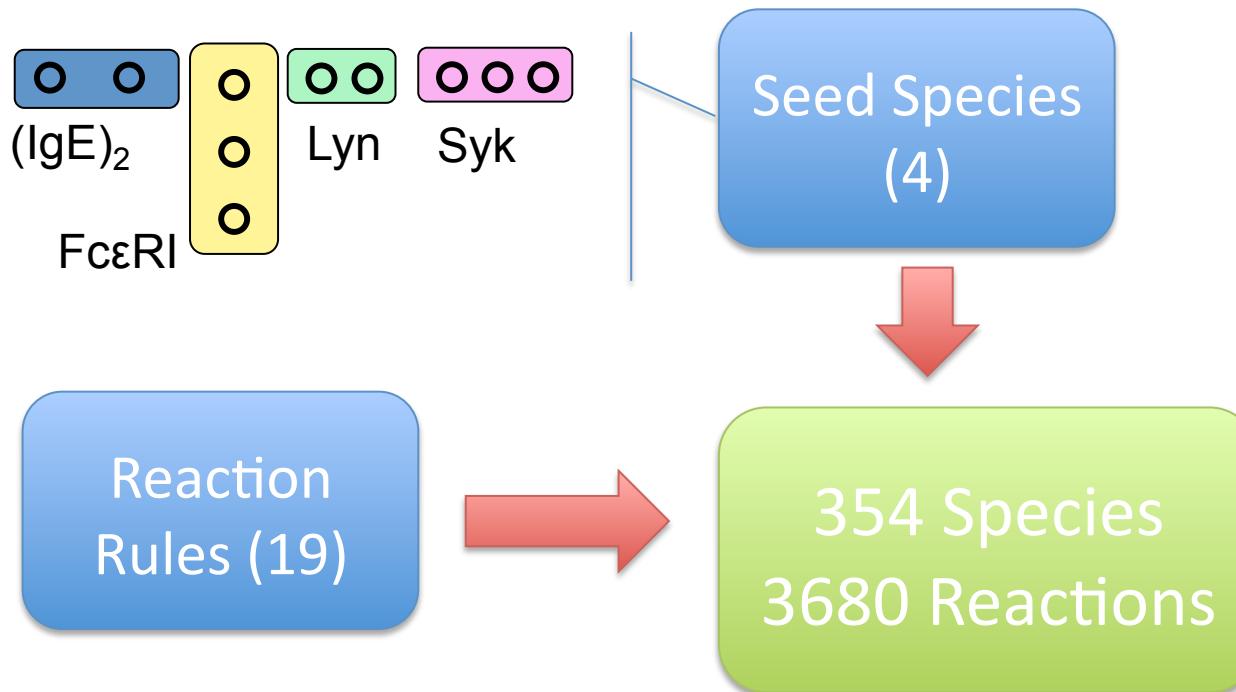
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Fc ϵ RI Model



Automatic Network Generation

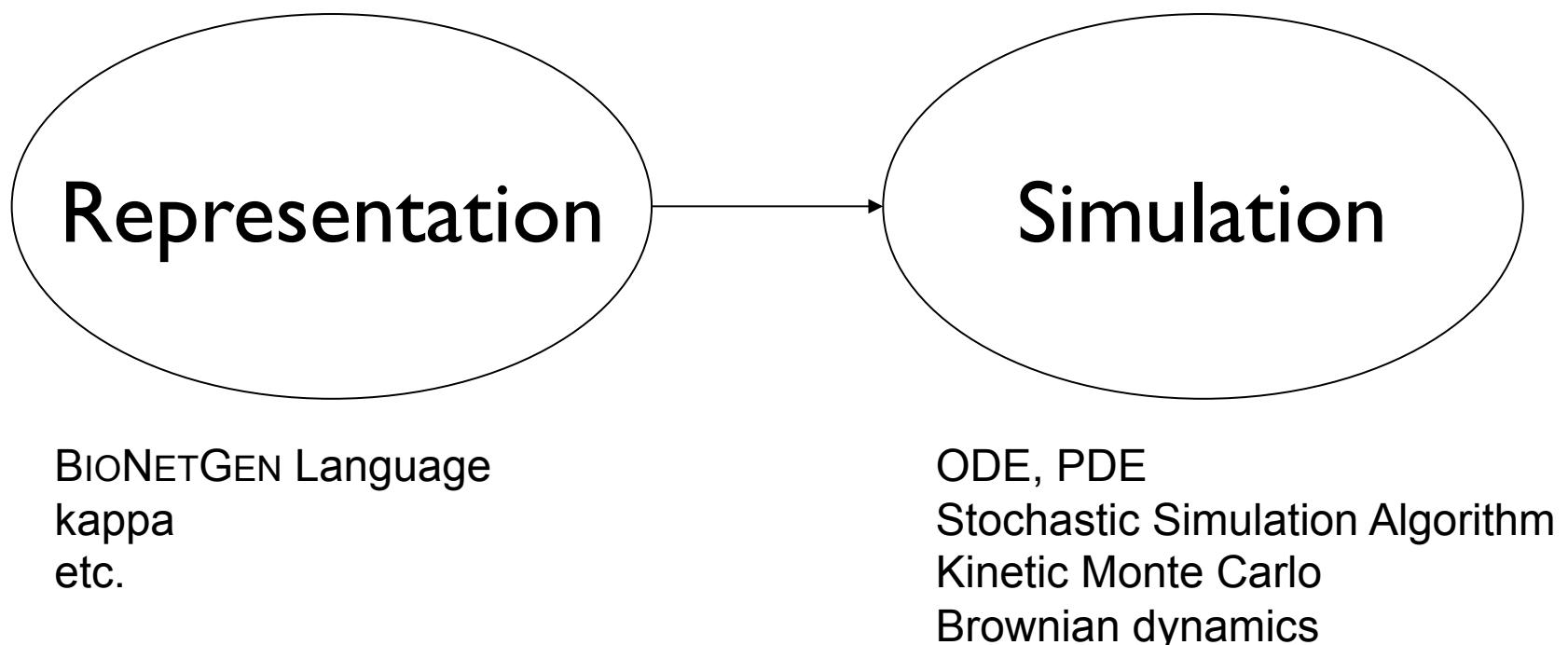
Fc ϵ RI Model



$$N_{\text{parameters}} \propto (N_{\text{rules}} + N_{\text{seed species}}) \ll N_{\text{reactions}}$$

Modeling cell signaling

AIM: Model the biochemical machinery by which cells process information (and respond to it).



Advantages of Formal Representations

- Precise interaction-based language for biochemistry – knowledge representation
- Concise representation of combinatorially complex systems
- Documentation and model readability
- Modularity and reusability
- Accuracy and rigor

Related Work

- StochSim
- Moleculizer
- Simmune
- κ -calculus / κ -factory
- little b
- Stochastic Simulation Compiler
- meredys
- ...

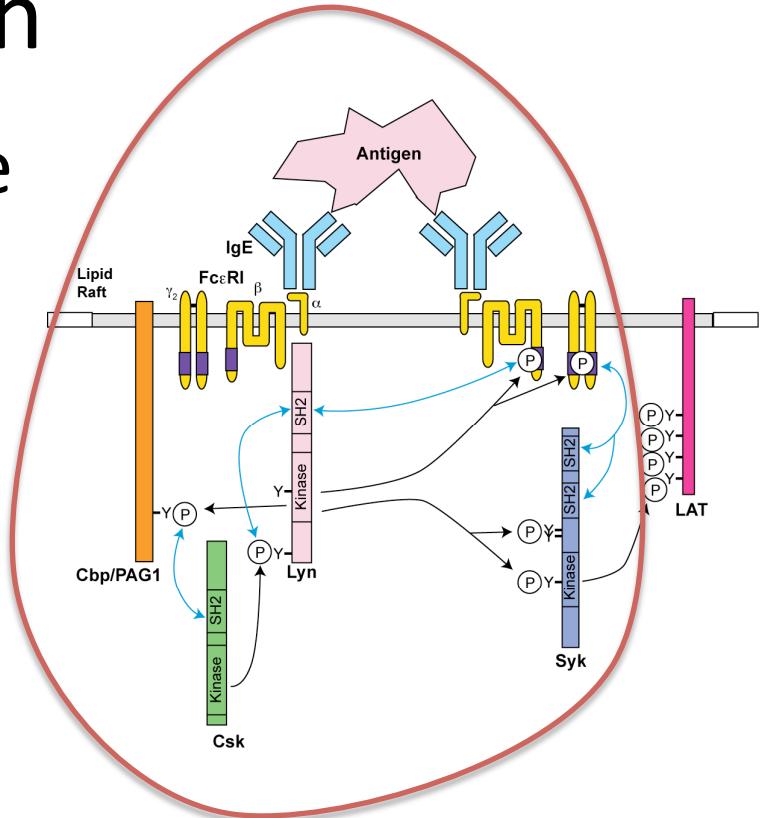
Systems Modeled

- IgE Receptor (Fc ϵ RI)
 - Faeder et al. *J. Immunol.* (2003)
 - Goldstein et al. *Nat. Rev. Immunol.* (2004)
 - Torigoe et al., *J. Immunol.* (2007)
 - Nag et al., *Biophys. J.*, (2009) [LAT]
- Receptor aggregation
 - Yang et al., *Phys. Rev. E* (2008)
- Growth Factor Receptors, other
 - Blinov et al. *Biosyst.* (2006) [EGFR]
 - Barua et al. *Biophys. J.* (2006) [Shp2]
 - Barua et al. *J. Biol. Chem.* (2008) [PI3K]
 - Barua et al., *PLoS Comp. Biol* (2009). [GH / SH2B]
- Carbon Fate Maps
 - Mu et al., *Bioinformatics* (2007)
- TCR (Lipniacki, *J. Theor. Biol.*, 2008)
- TLR4 (An & Faeder, *Math. Biosci.*, 2009)

See <http://bionetgen.org> for complete list.

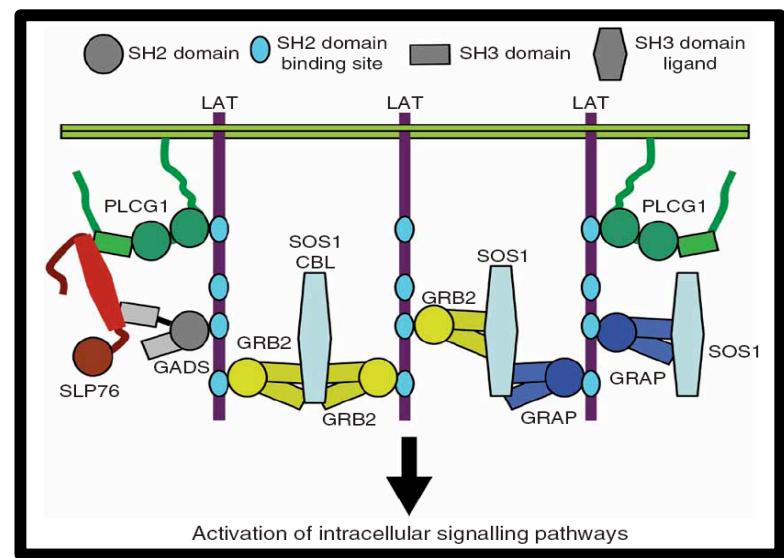
Limits of the network generation approach

- Extending model to include Lyn regulation results in >20,000 states.



Limits of the network generation approach

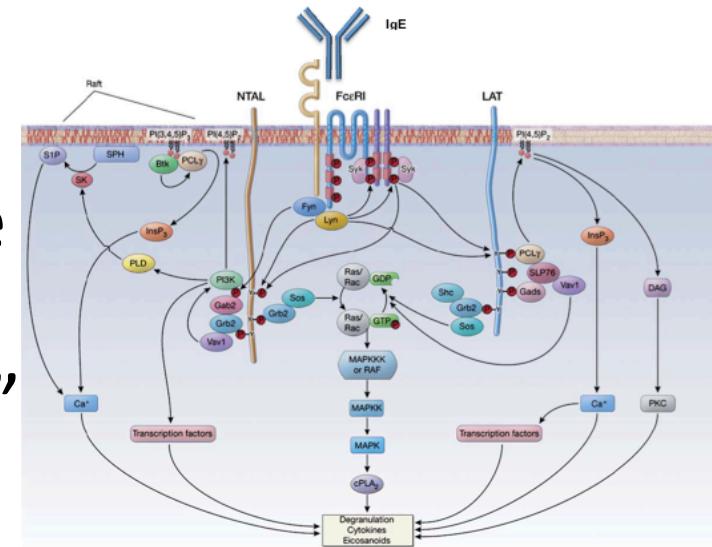
- Extending model to include Lyn regulation results in >20,000 states.
- LAT may form large oligomers under physiological conditions.



Houtman et al., *Nat. Struct. Mol. Biol.* (2006)
Nag et al., *Biophys. J.* (2009)

Limits of the network generation approach

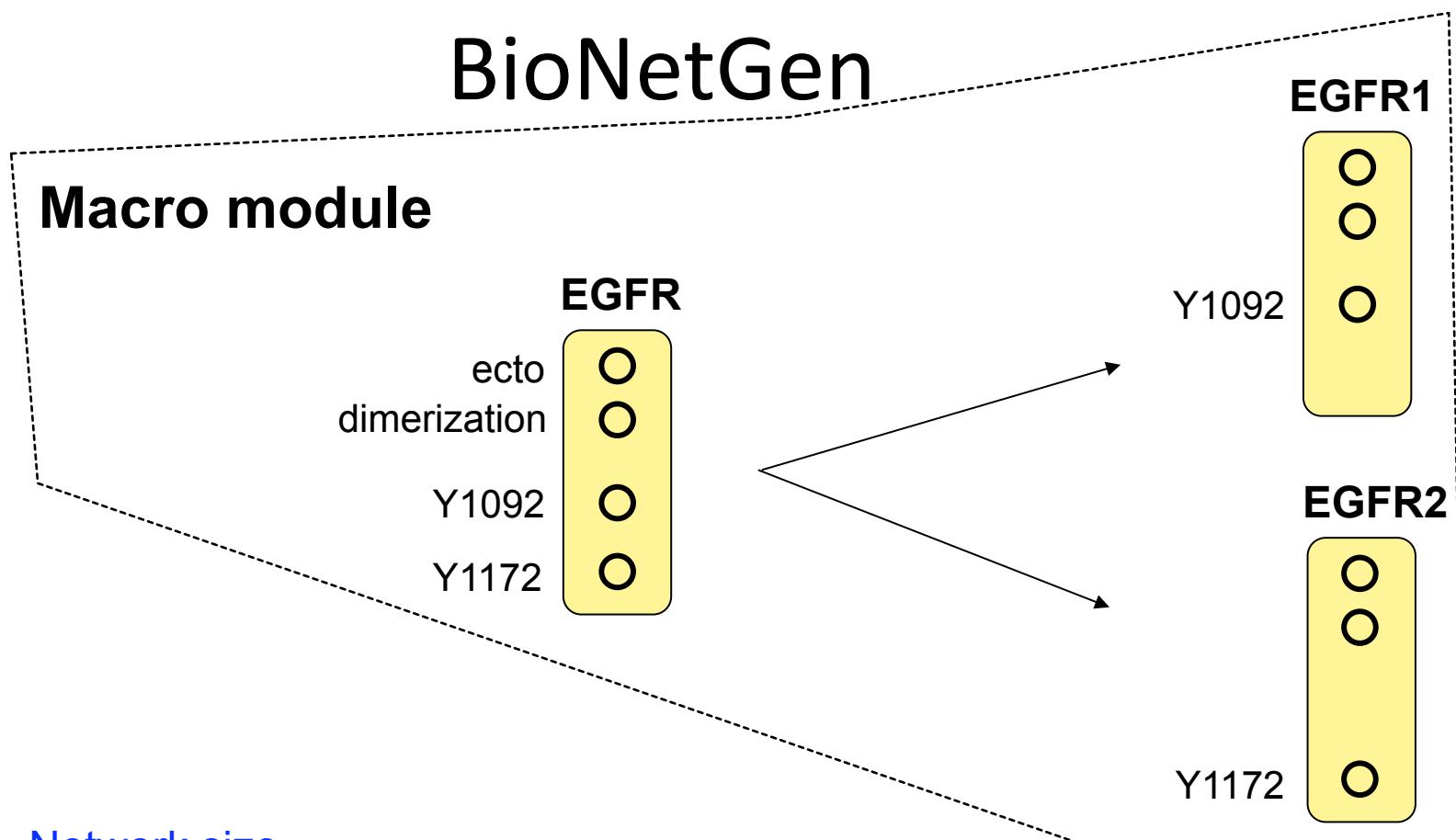
- Extending model to include Lyn regulation results in >20,000 states.
- LAT may form large oligomers under physiological conditions.
- Many more components are still missing. Networks can easily reach “Avogadro limit”



Handling the combinatorial explosion

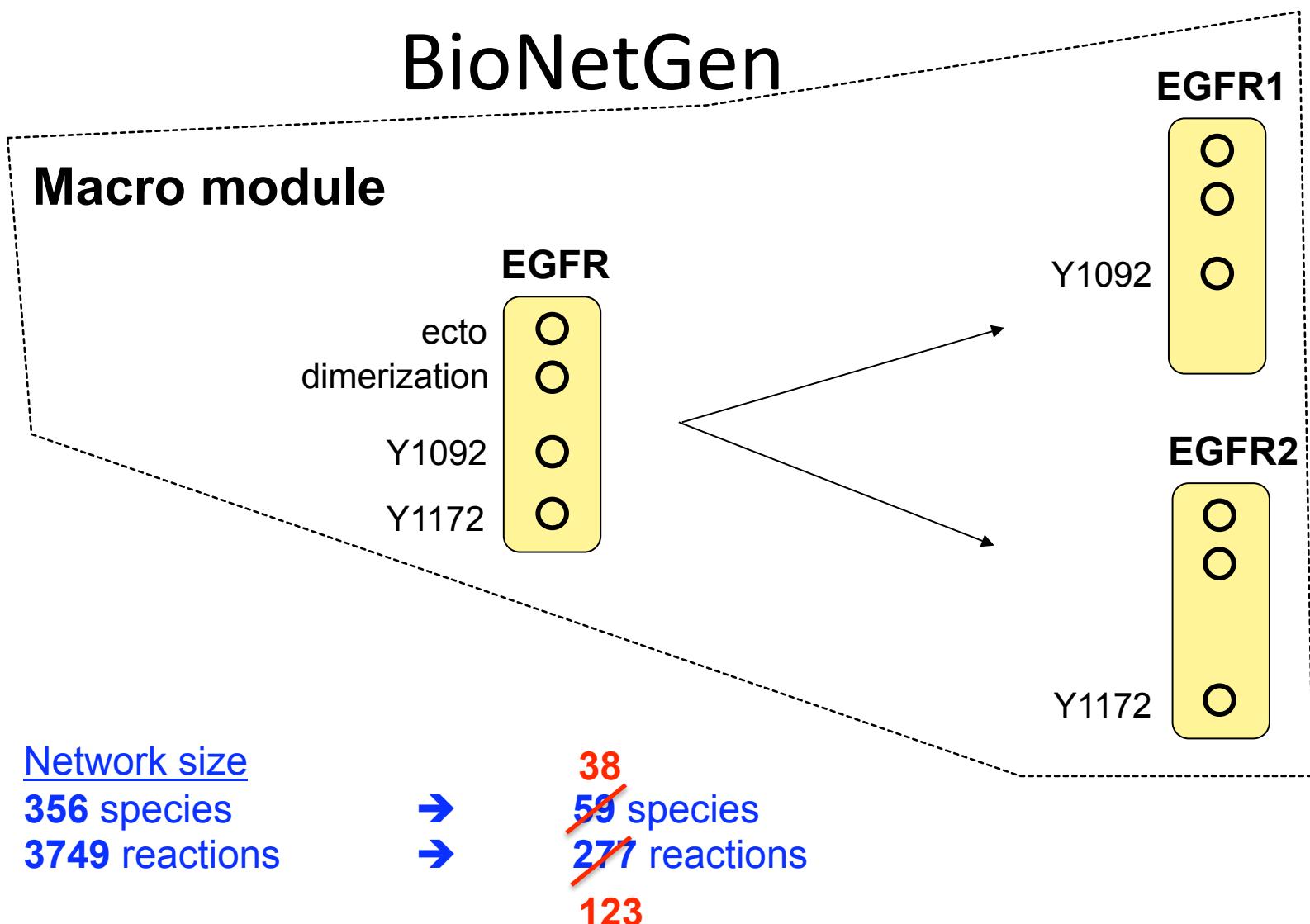
- **Model truncation**
 - Arbitrary
 - Loss of accuracy / convergence check
- **Model reduction**
 - Applies when sites act independently (multiplicative → additive)
 - Limited by cooperativity (FcεRI model is not reducible)
 - See Feret et al. *PNAS* (2009); Conzelmann et al. *BMC Syst. Biol.* (2008); Borisov et al., *IET Syst. Biol.* (2008)

Automated model reduction in BioNetGen



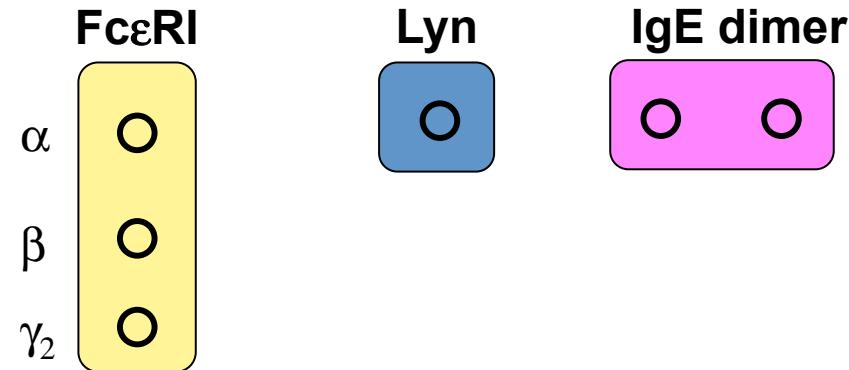
Borisov et al., *IET Syst. Biol.* (2008)

Automated model reduction in BioNetGen

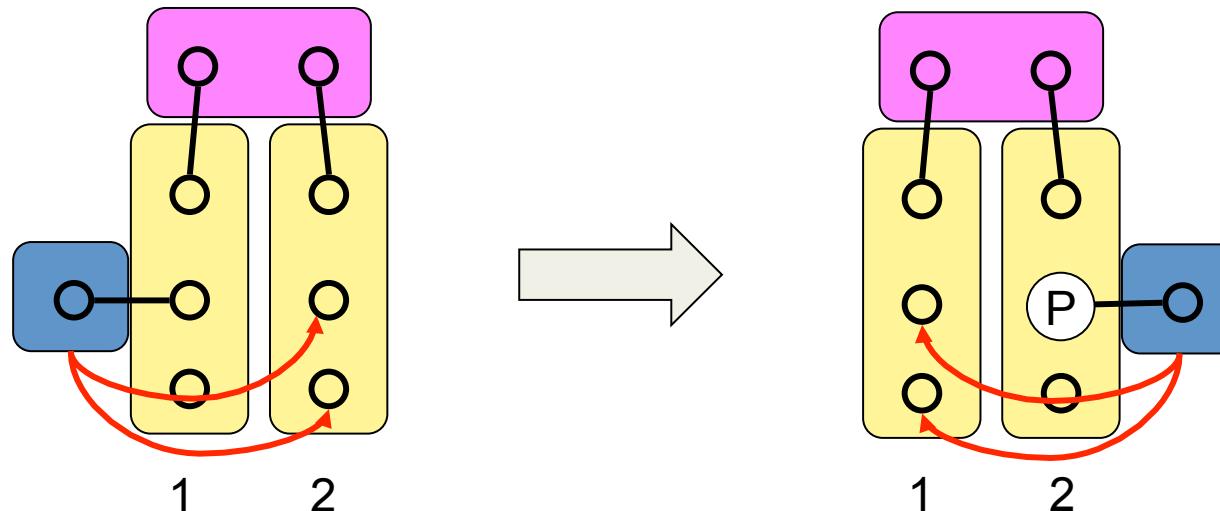


Conzleman, Faeder, and Hogg, unpublished.

Cooperativity leads to irreducibility

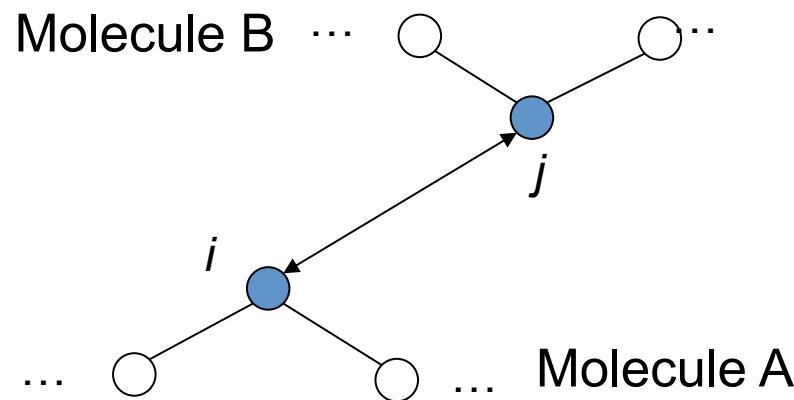


Transphosphorylation by Lyn couples sites



Interaction-based formulation of chemical kinetics

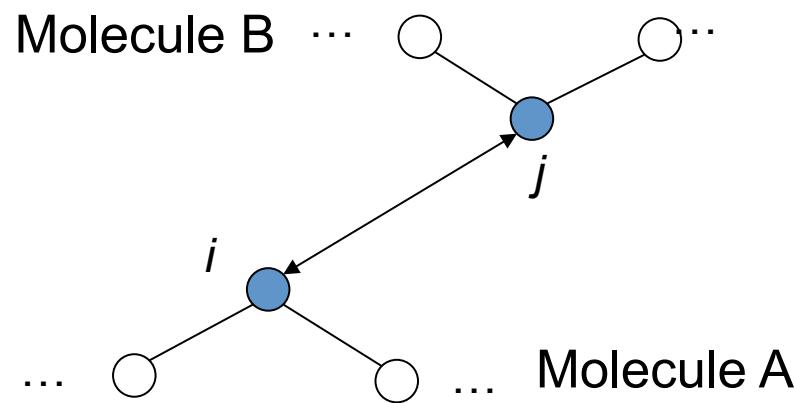
Molecular Dynamics (MD)



Dynamics depend on force
between i and j (Newton)

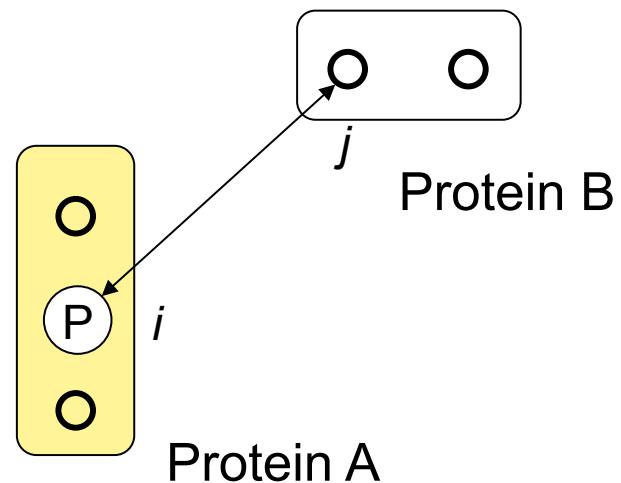
Interaction-based formulation of chemical kinetics

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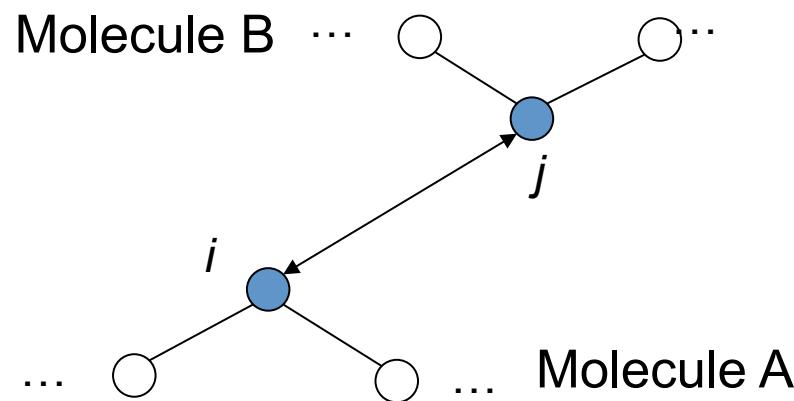
Chemical Kinetics



Dynamics depend on reaction
propensity between i and j (Gillespie)

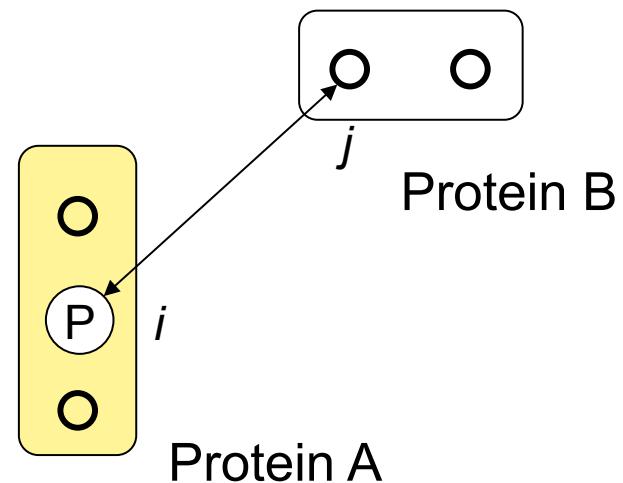
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Chemical Kinetics

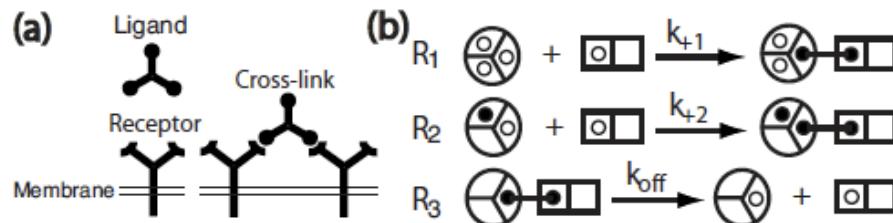


Dynamics depend on reaction
propensity between i and j (Gillespie)

Key point: Dynamics depend on time-varying quantities that may be computed on-the-fly. *Don't have to enumerate possibilities in advance.*

Kinetic Monte Carlo method for rule-based modeling of biochemical networks

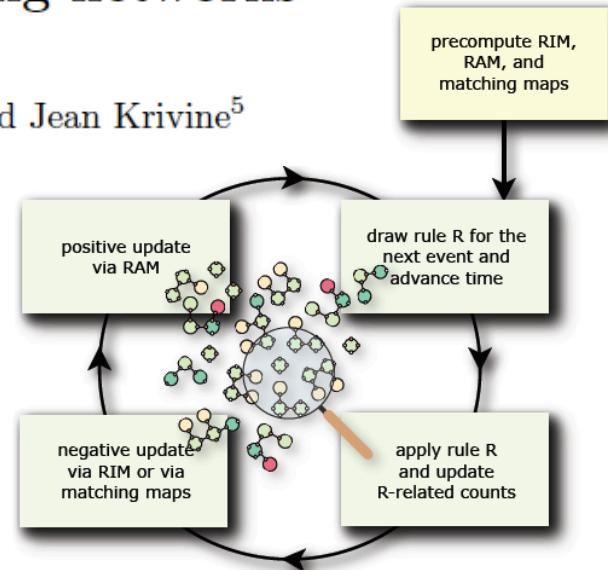
Jin Yang,^{1,*} Michael I. Monine,² James R. Faeder,^{3,†} and William S. Hlavacek^{2,‡}



Scalable simulation of cellular signaling networks

Vincent Danos^{1,4*}, Jérôme Feret³, Walter Fontana^{1,2}, and Jean Krivine⁵

APLAS '07 (invited paper)



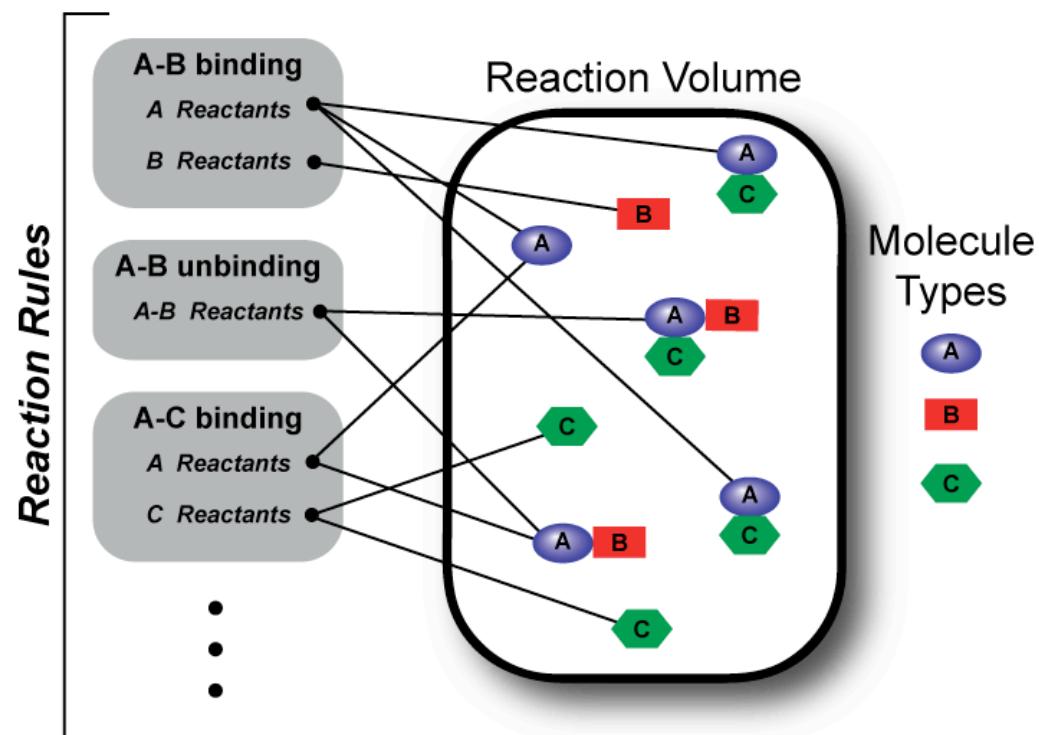
NFsim

The Network-Free Stochastic Simulator

- 1) Treat molecules as Agents that can be connected together.
- 2) Use reaction rules to define interactions.
- 3) Simulate with an Agent-based extension to the Gillespie Algorithm



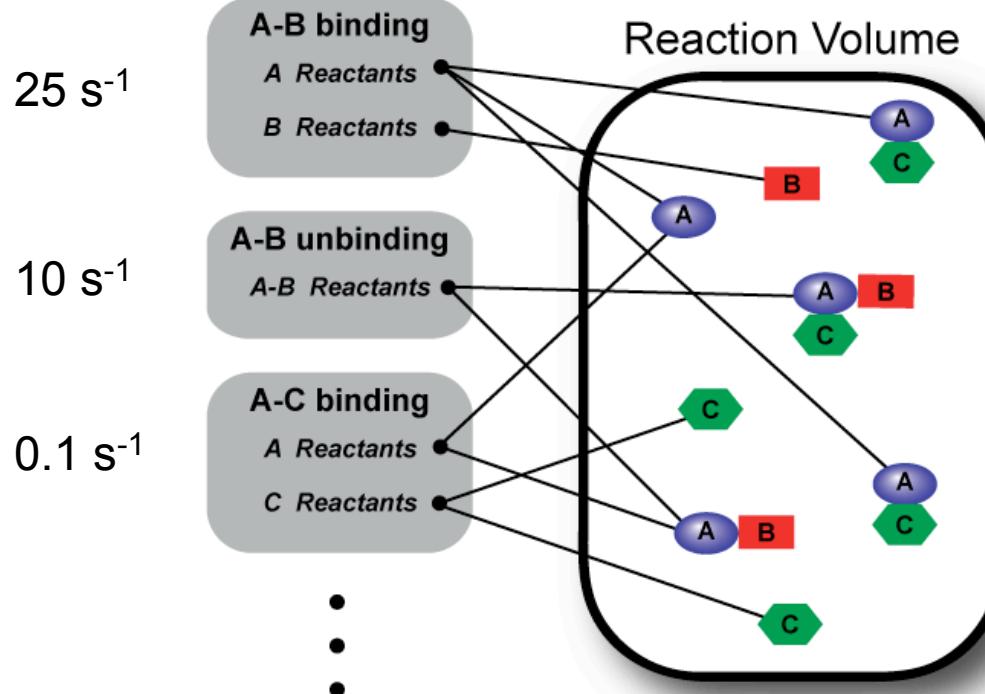
Michael Sneddon
Thierry Emonet
Yale University



The Event Scheduler

An Agent-based Extension to the Gillespie Algorithm

Rates



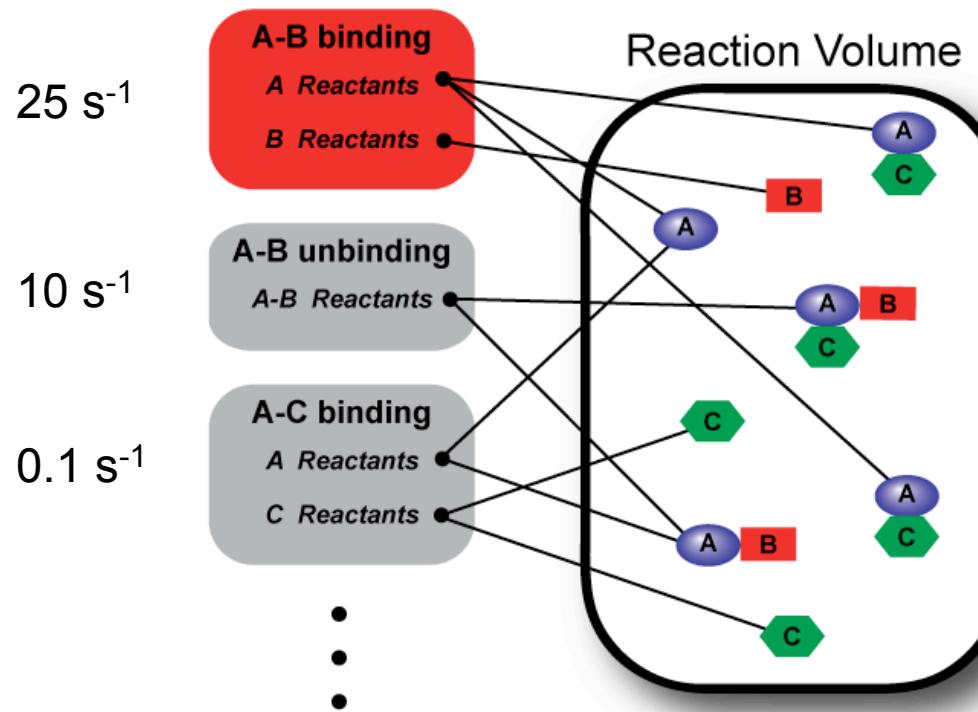
Pointers to possible reactants stored on Rule-Reactant lists.

Sneddon, Faeder, and Emonet, in preparation.

The Event Scheduler

An Agent-based Extension to the Gillespie Algorithm

Rates



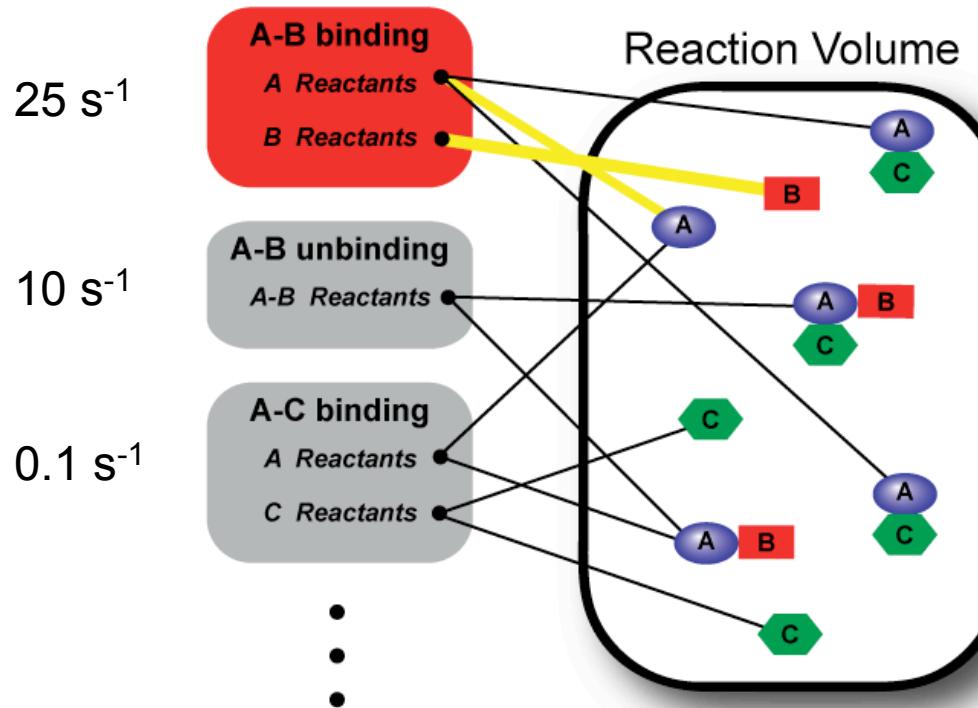
As in Gillespie, the waiting time to the next event is sampled.

Here, the next RULE is chosen stochastically (not the next reaction).

The Event Scheduler

An Agent-based Extension to the Gillespie Algorithm

Rates



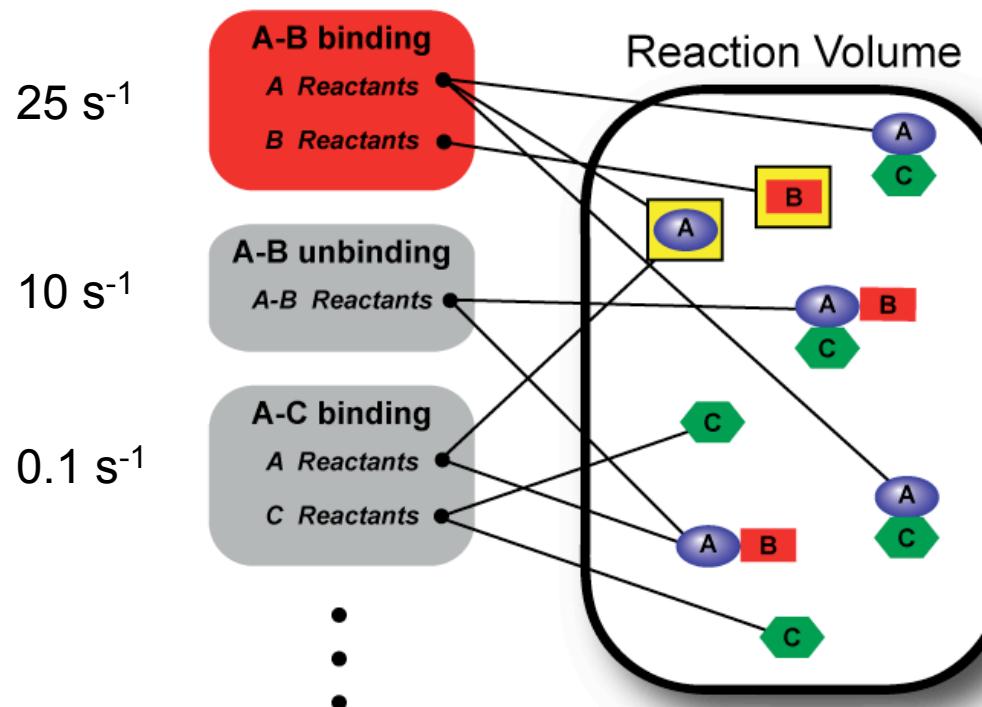
The molecule Agents that will react are randomly selected.

Sneddon, Faeder, and Emonet, in preparation.

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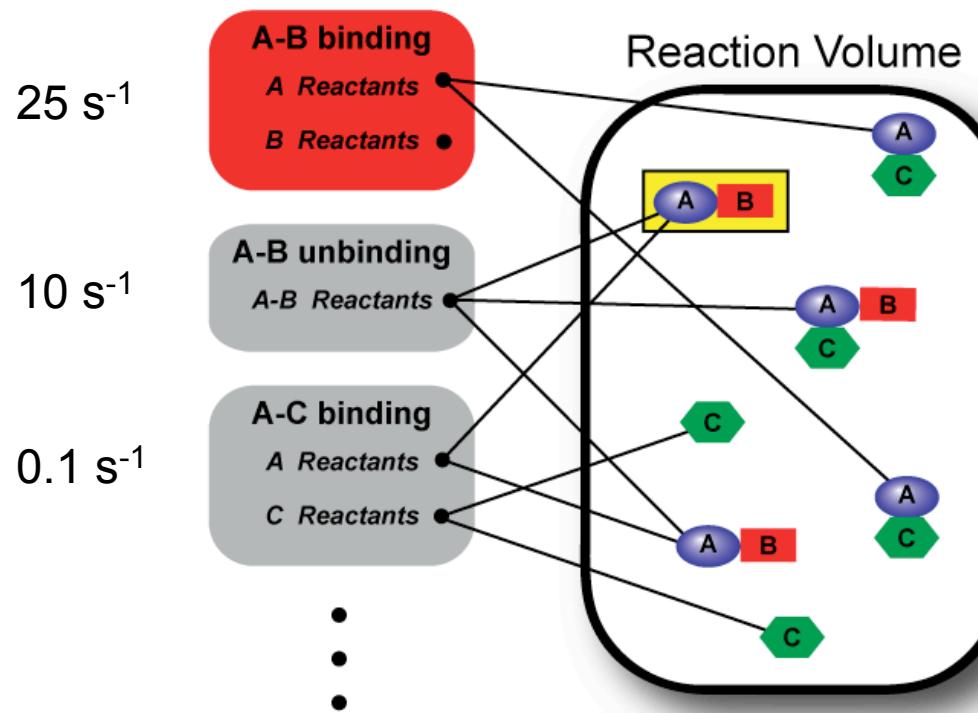


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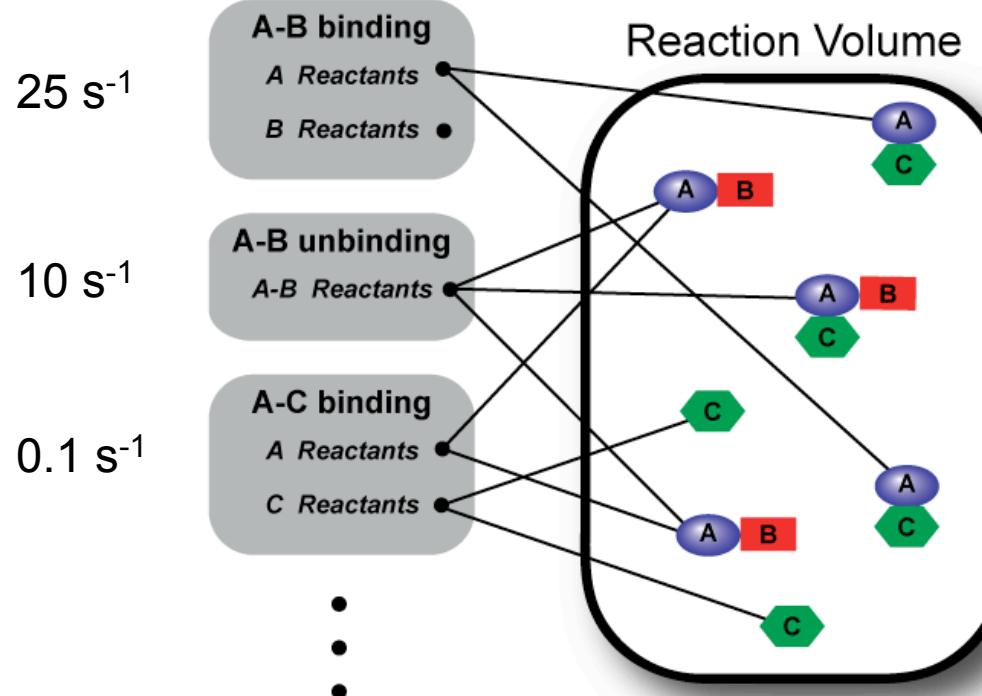


The molecule Agents update themselves and “reschedule” themselves by updating their Rule-Reactant Pointers.

The Event Scheduler

An Agent-based Extension to the Gillespie Algorithm

Rates

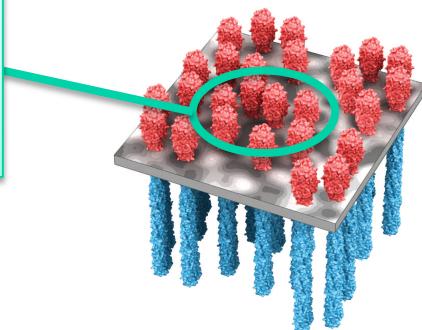


The system is advanced by the sampled time.

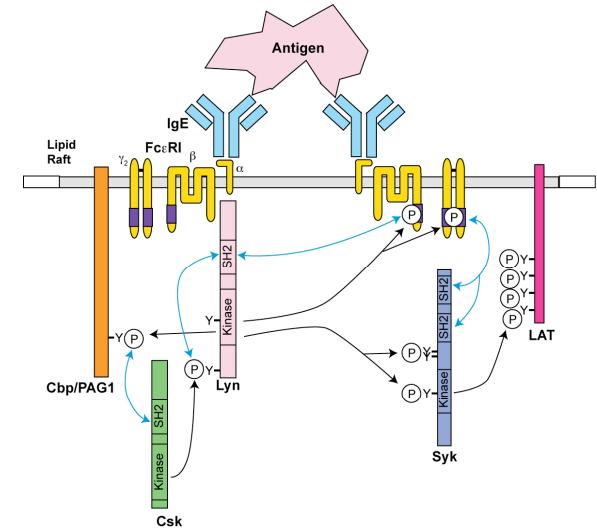
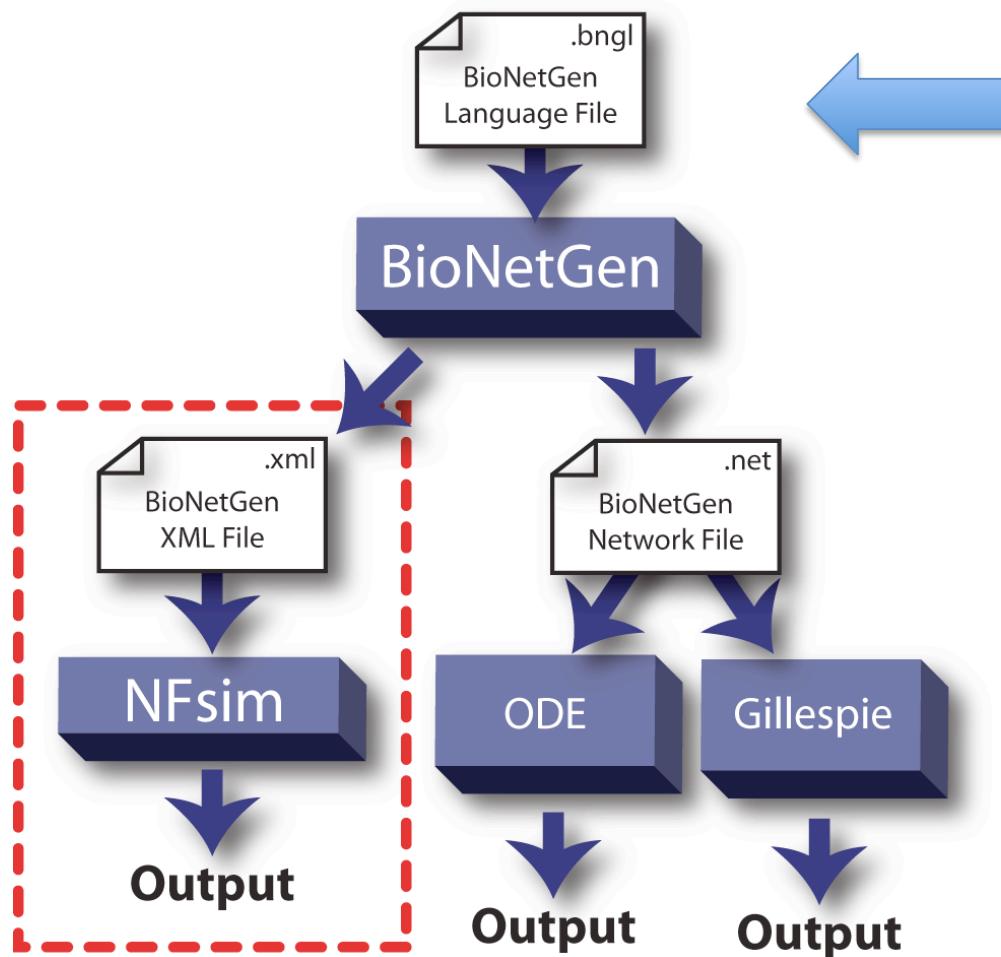
NFsim Core Simulator Features

- 1) Modular C++ code base and highly efficient implementation
- 2) Operates seamlessly with BioNETGEN
- 3) Extended BioNetGen Language handles
 - 1) Spatial compartments
 - 2) System variables in rate law expressions

cooperative
receptor
interactions


$$\text{MethLevel}(x) = 1*R1(x)+2*R2(x)+3*R3(x)+4*R4(x)+5*R5(x)+6*R6(x)+7*R7(x)+8*R8(x)$$

Integration with BioNetGEN



DYNSTOC: An Alternative Network-Free Approach Based on Null Event Sampling

Systems biology

Bioinformatics

Simulation of large-scale rule-based models

Joshua Colvin¹, Michael I. Monine², James R. Faeder³, William S. Hlavacek^{2,4}, Daniel D. Von Hoff⁵, and Richard G. Posner^{1,6*}

NFsim: Event-driven, slower updates

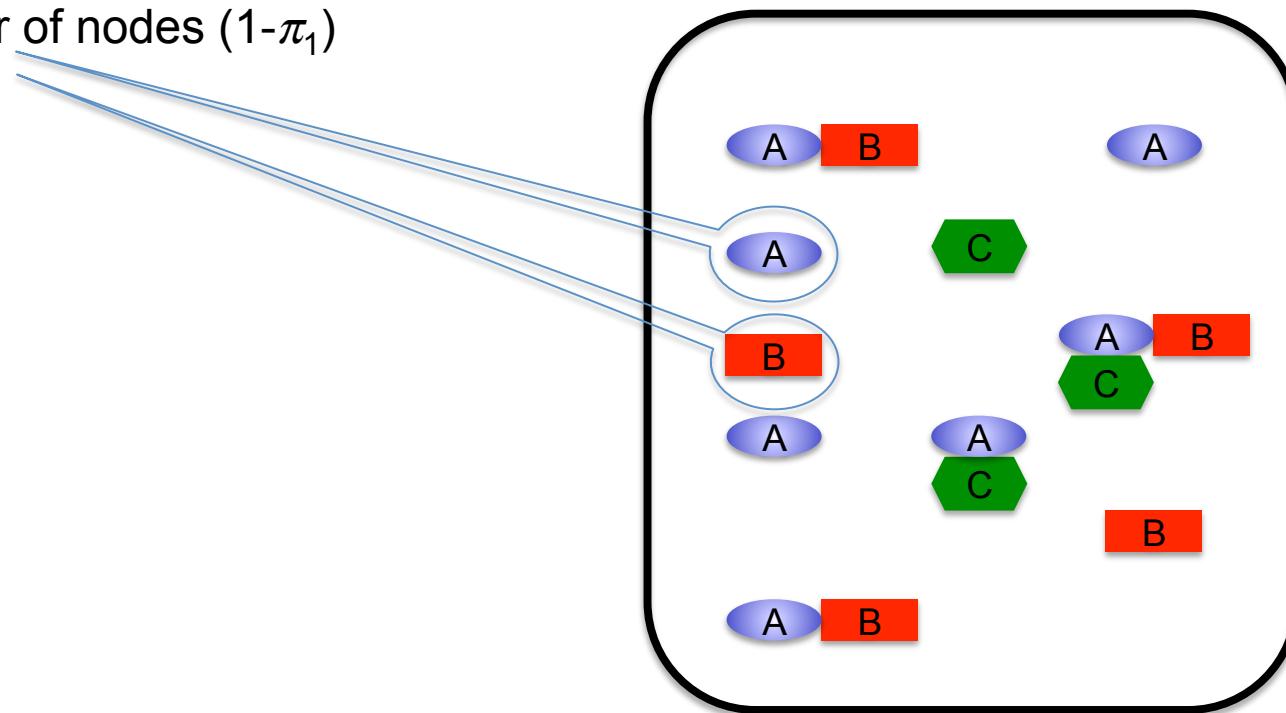


StochSim: Fixed time step, fast updates

(Morton-Firth and Bray, 1998)

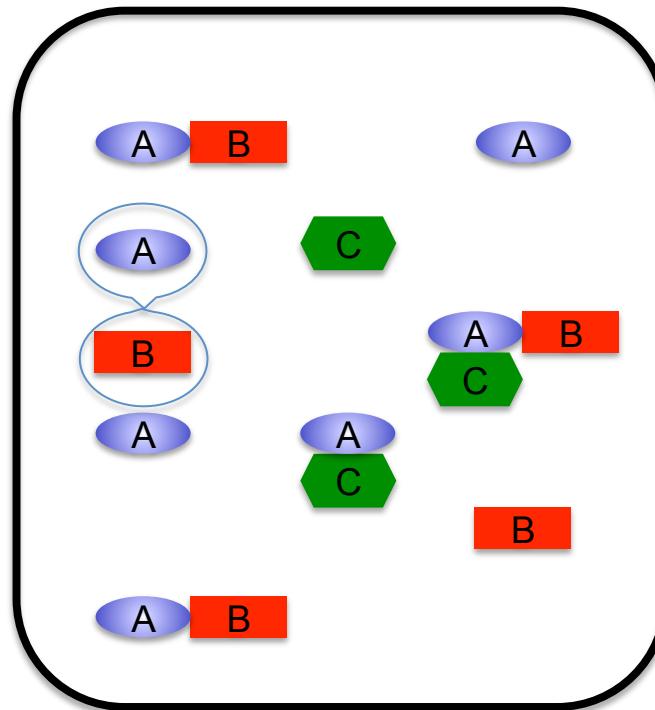
DYNSTOC Algorithm

1. Pick a node (π_1) or pair of nodes ($1-\pi_1$)



DYNSTOC Algorithm

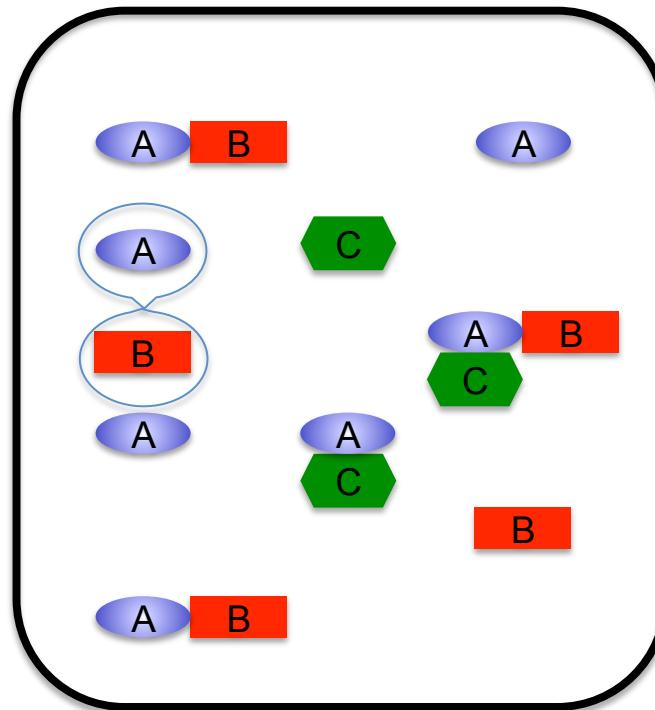
1. Pick a node (π_1) or pair of nodes ($1-\pi_1$)
2. Compute reaction probabilities (p_i) for Δt .



DYNSTOC Algorithm

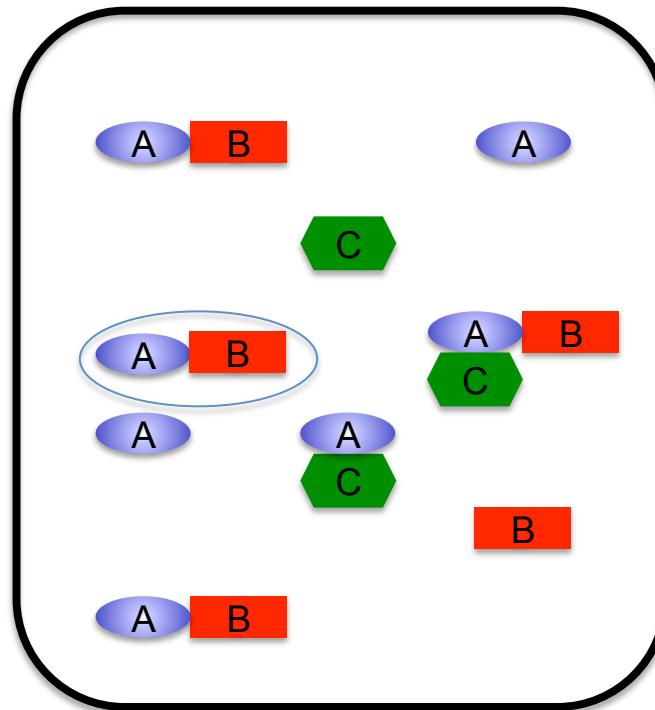
1. Pick a node (π_1) or pair of nodes ($1-\pi_1$)
2. Compute reaction probabilities (p_i) for Δt .
3. Determine if a reaction occurs.

smallest R s.t. $\sum_{i=1}^R p_i \geq \rho \in (0,1)$



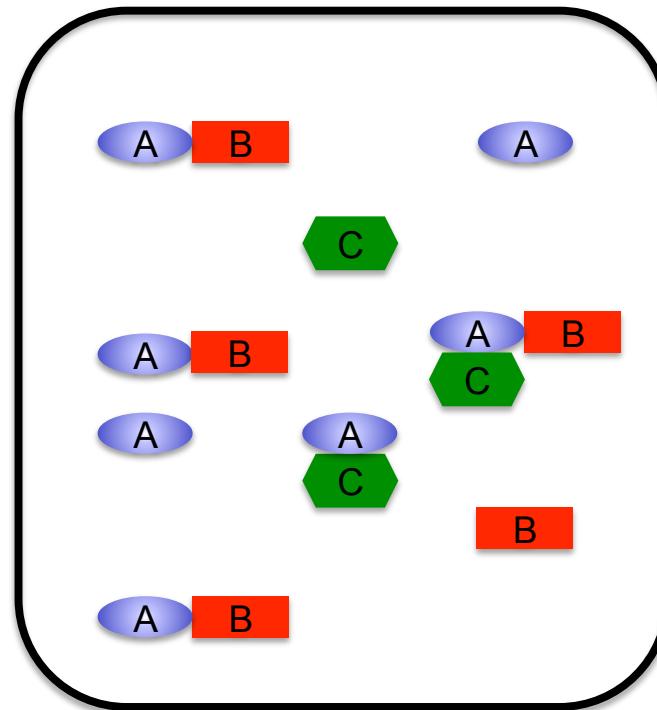
DYNSTOC Algorithm

1. Pick a node (π_1) or pair of nodes ($1-\pi_1$)
2. Compute reaction probabilities (p_i) for Δt .
3. Determine if a reaction occurs.
4. If yes, fire reaction; otherwise, *null* event.



DYNSTOC Algorithm

1. Pick a node (π_1) or pair of nodes ($1-\pi_1$)
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3. Determine if a reaction occurs.
4. If yes, fire reaction; otherwise, *null* event.
5. Increment time:
$$t = t + \Delta t$$



Trivalent Ligand Bivalent Receptor (TLBR)

Goldstein and Perelson (1984)

Ligand



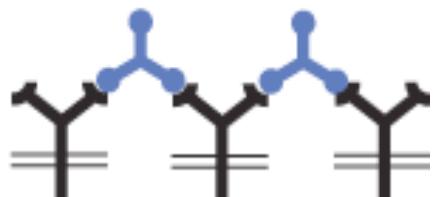
Receptor



Membrane

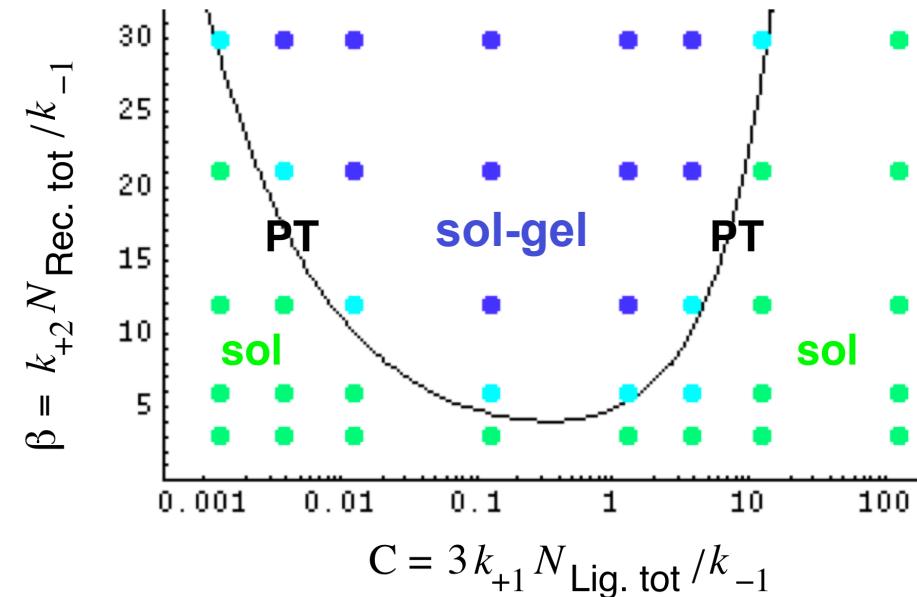
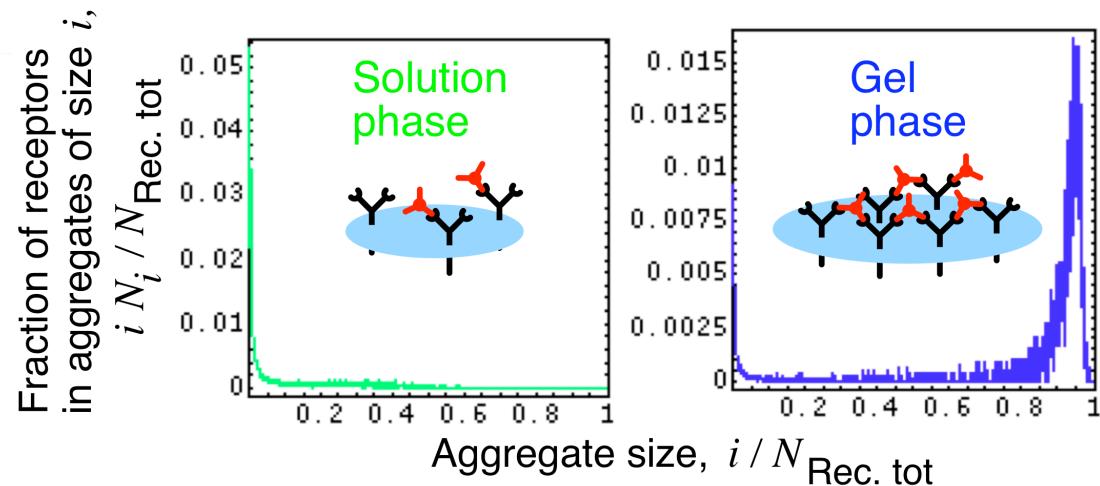
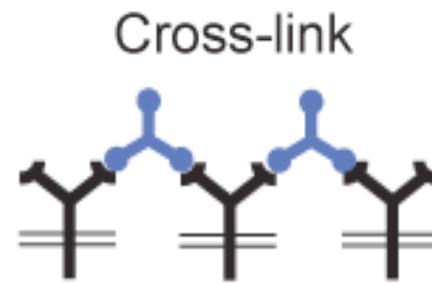
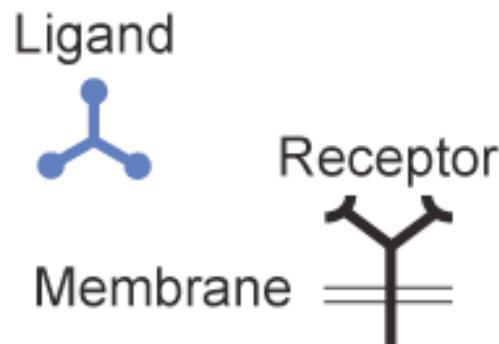


Cross-link



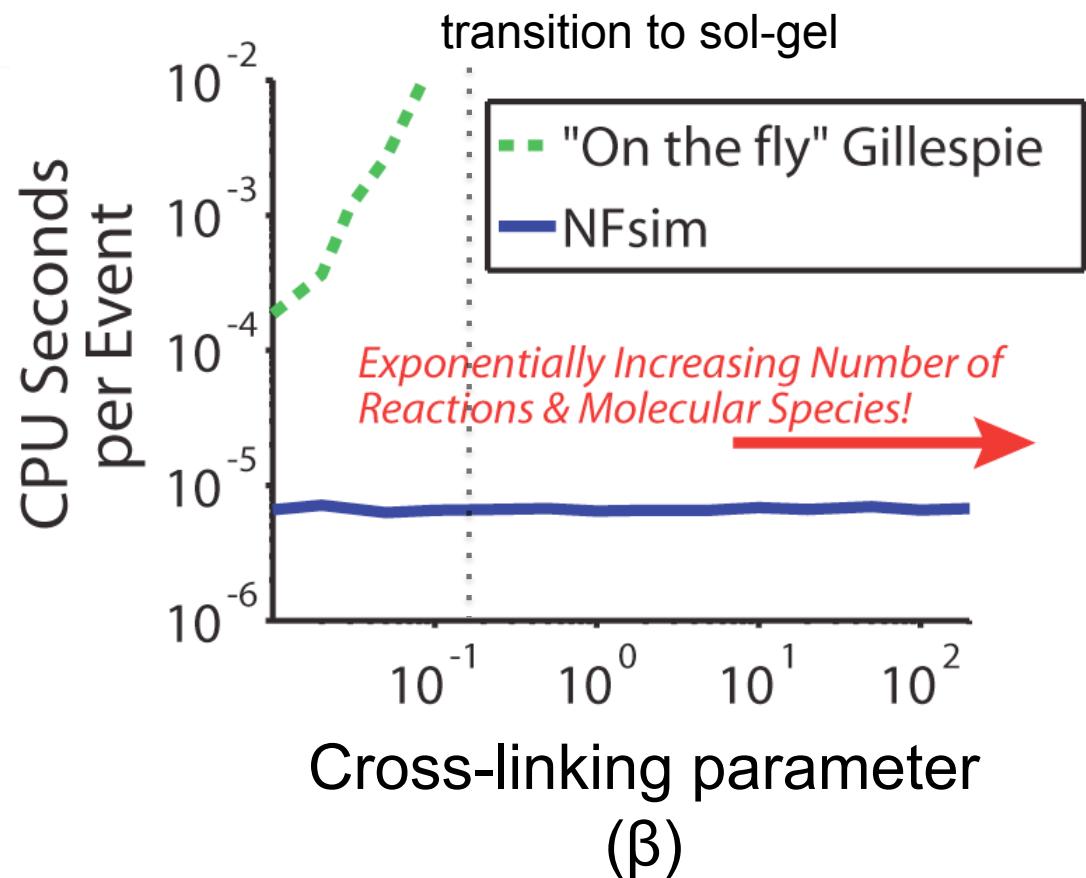
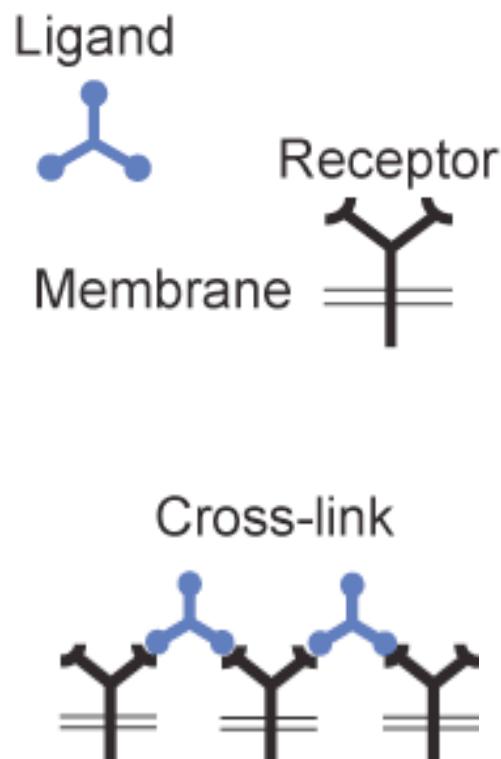
Trivalent Ligand Bivalent Receptor (TLBR)

Goldstein and Perelson (1984)



NFsim's Performance

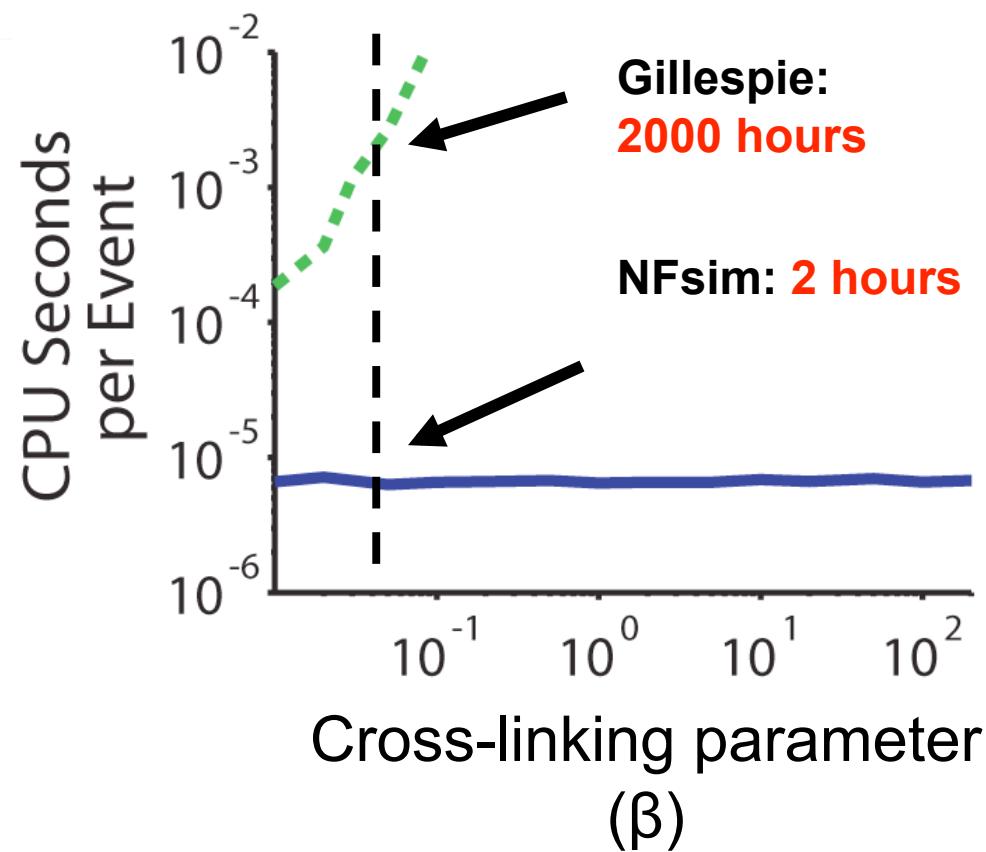
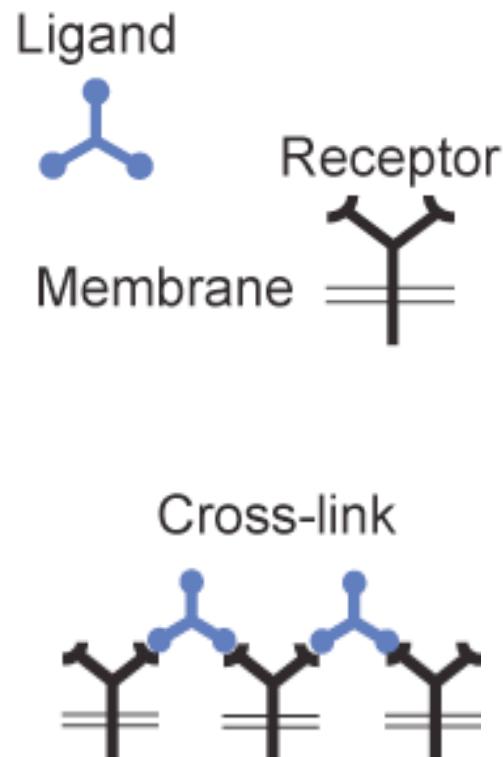
Trivalent Ligand, Bivalent Receptor (TLBR) System



3000 Receptors, 10,000 Ligands

NFsim's Performance

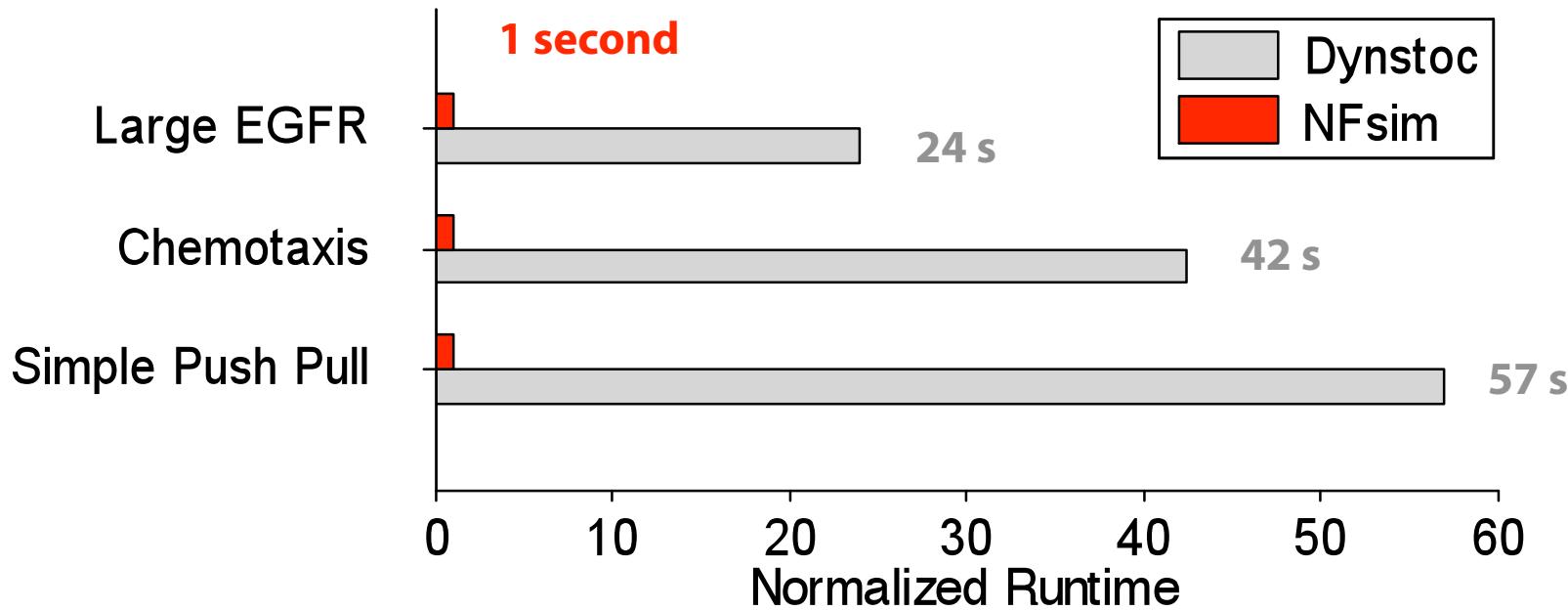
Trivalent Ligand, Bivalent Receptor (TLBR) System



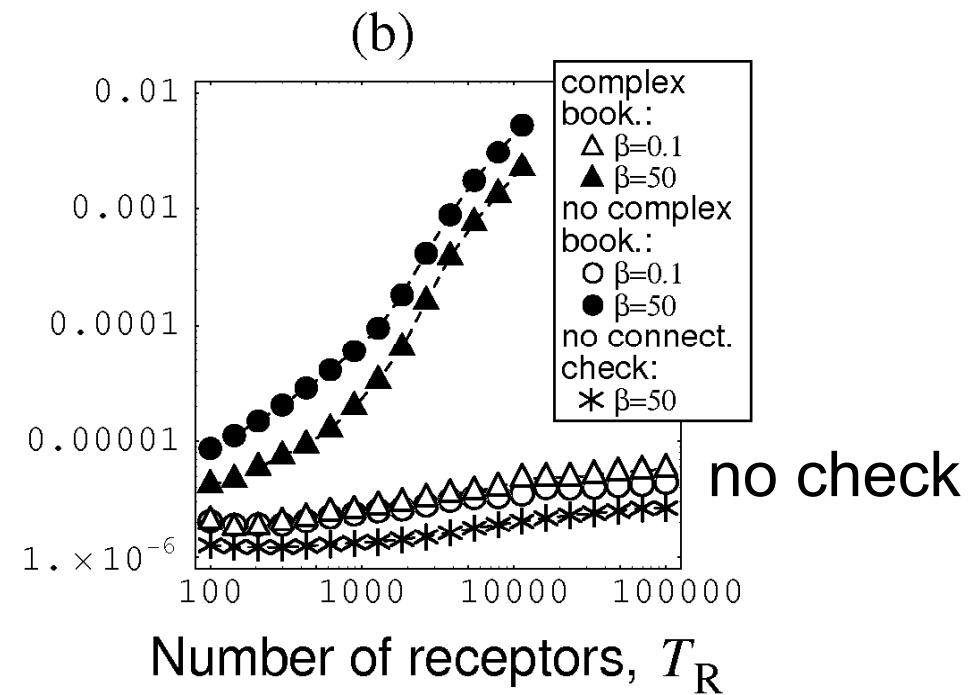
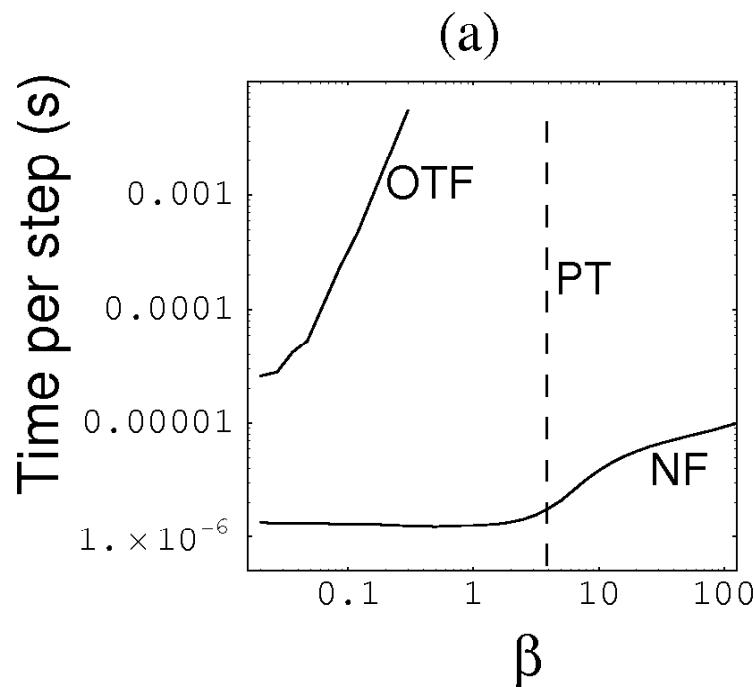
3000 Receptors, 10,000 Ligands

NFsim's Performance

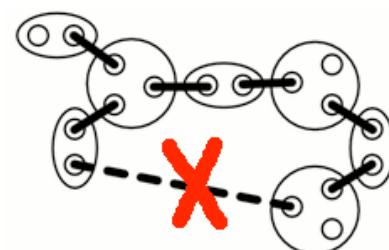
Comparison to the StochSim / Dynstoc Approach



Performance

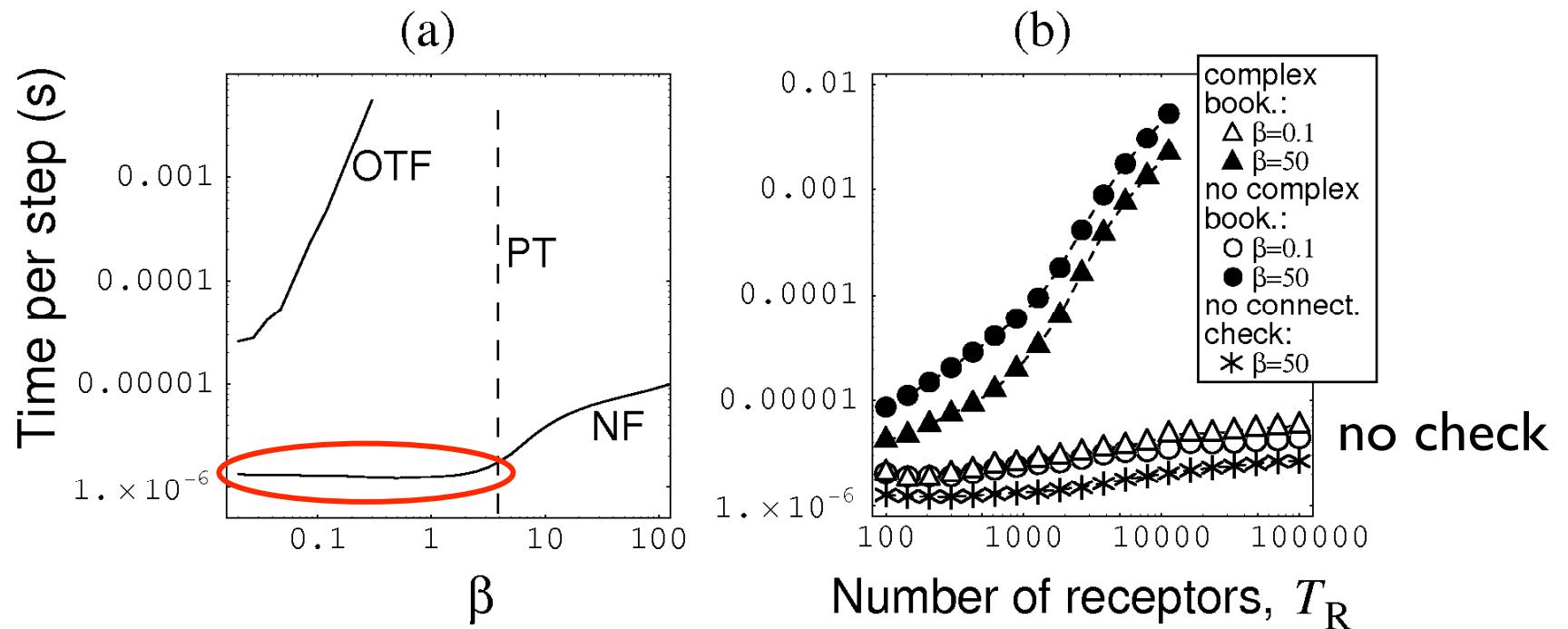


Modest increase in cost
at gel phase boundary
due to connectivity check



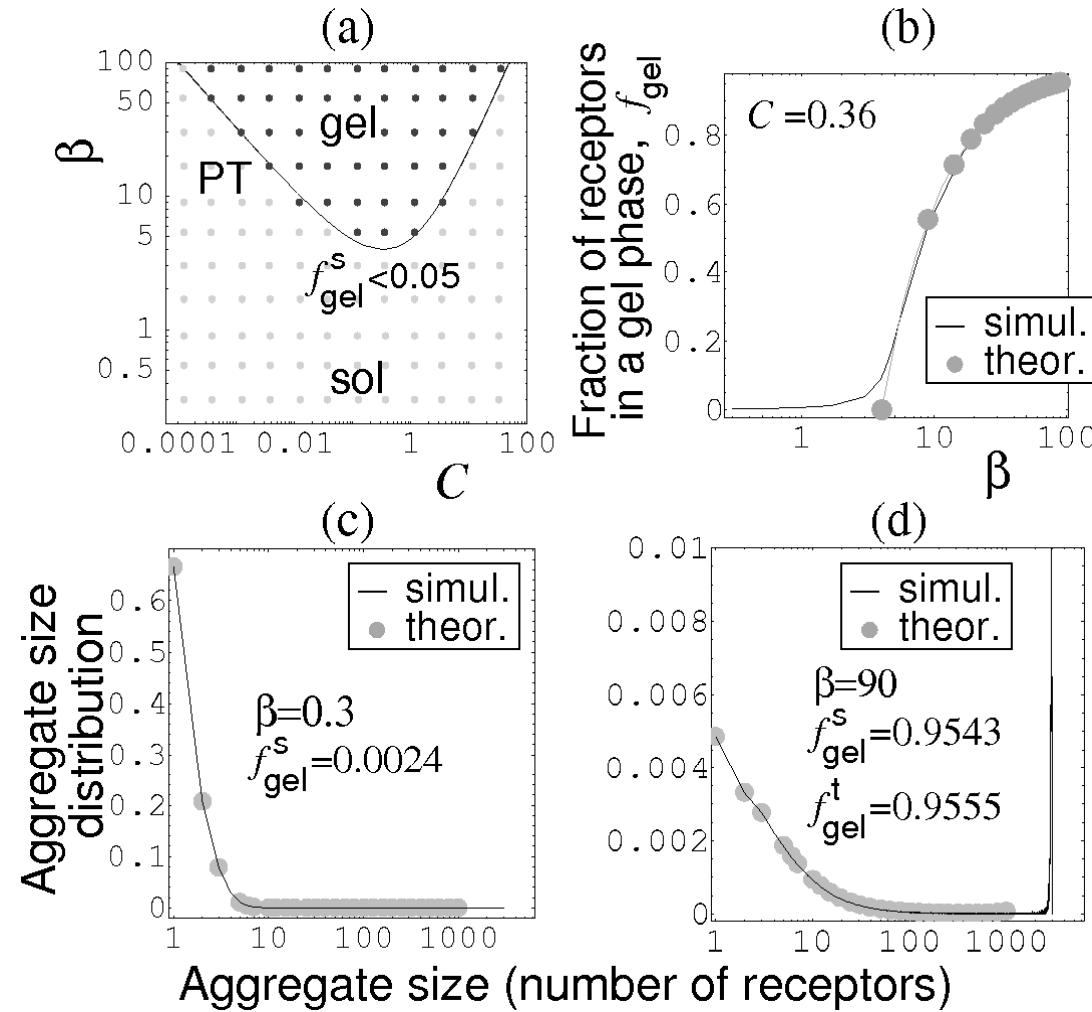
Michael Monine

Performance



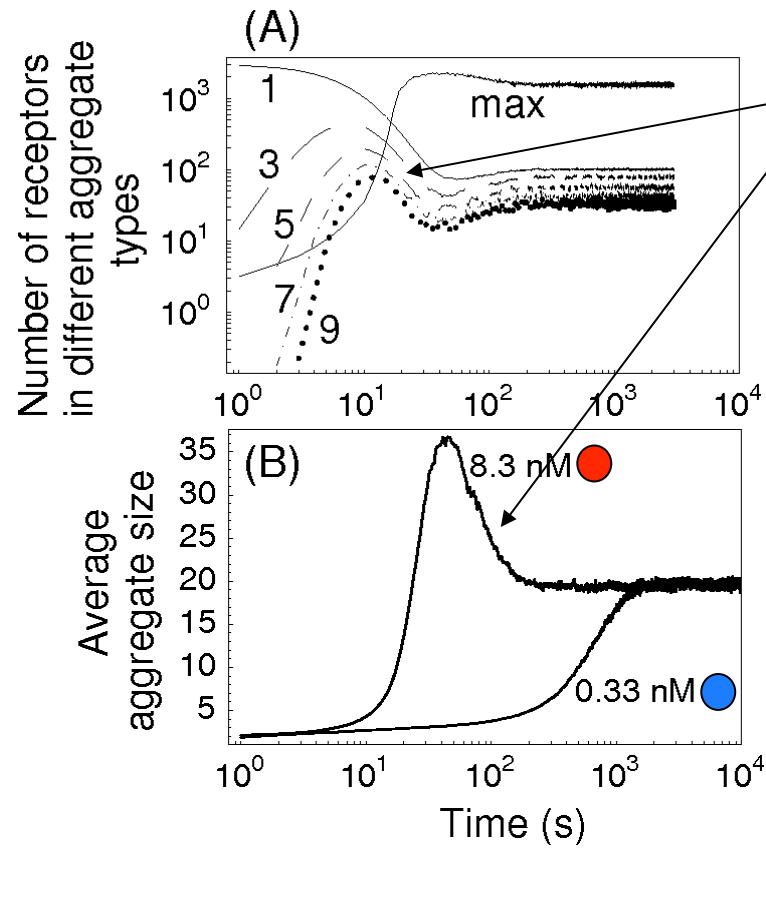
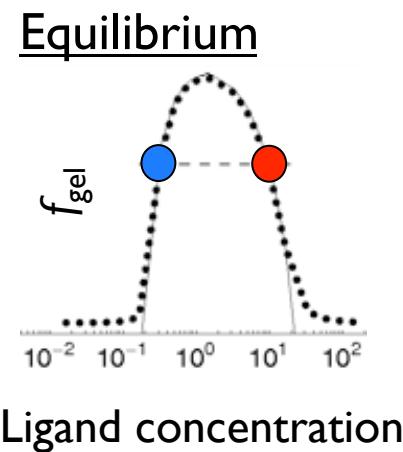
Network size increases
dramatically, but cost is
constant

Validation based on comparison with Goldstein-Perelson Theory



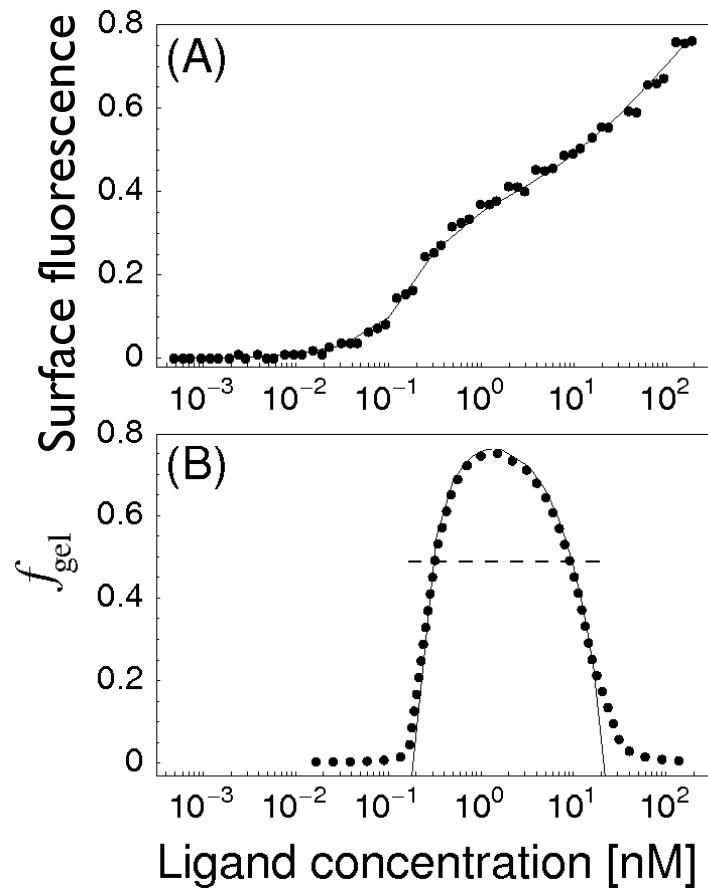
Monine et al., submitted

Kinetics of Aggregate Formation



At high ligand concentration, intermediate aggregates form transiently

Trivalent DNP ligand binding to IgE-Fc ϵ RI

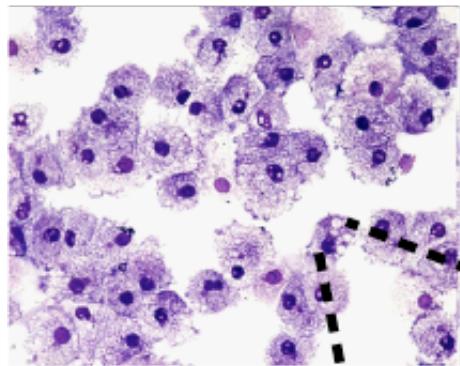


Flow cytometry data from R. Posner (NAU/TGen) for fluorescein-labeled trivalent DNP ligand.

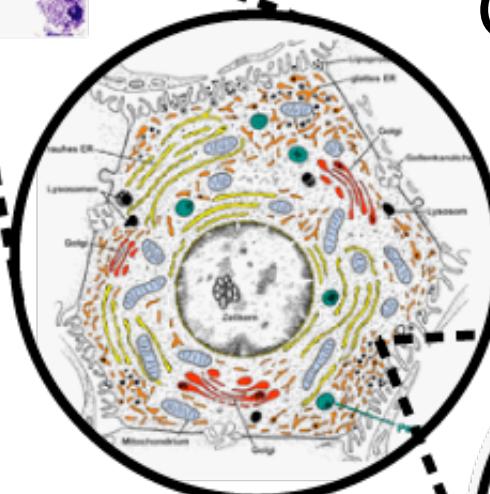
Fit model parameters lie within gel formation region

Finding is robust to inclusion of steric effects and ring formation in model

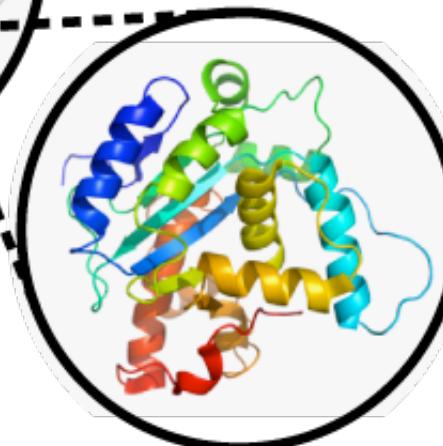
Agents at all Scales of Biology



Cells as Agents



Organelles as
Agents



Molecules
as Agents

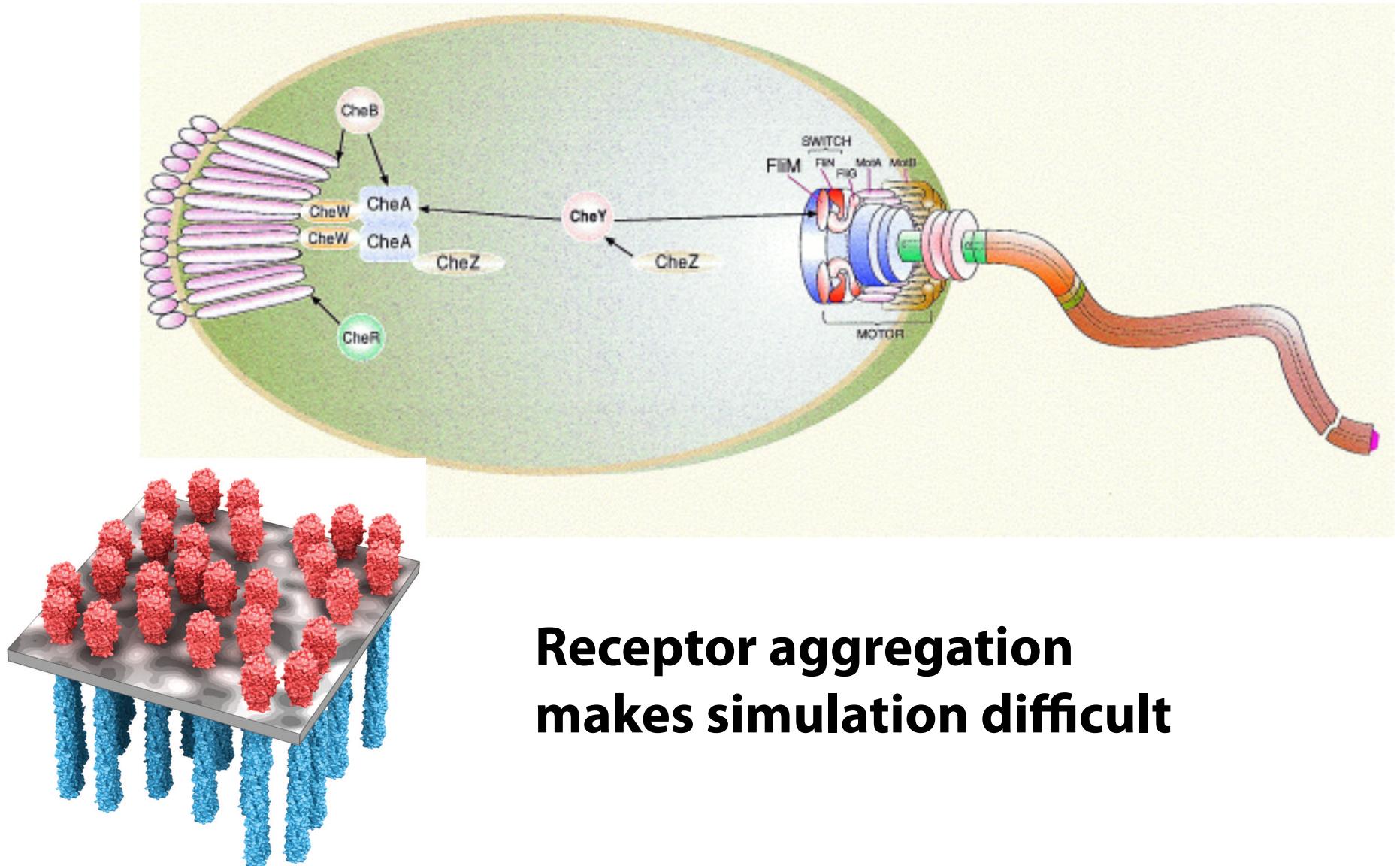
Goal: Multiscale
Agent-based, simulation
of biological systems,
building up from the
stochastic molecular
level

Cell and Population
Level Behavior

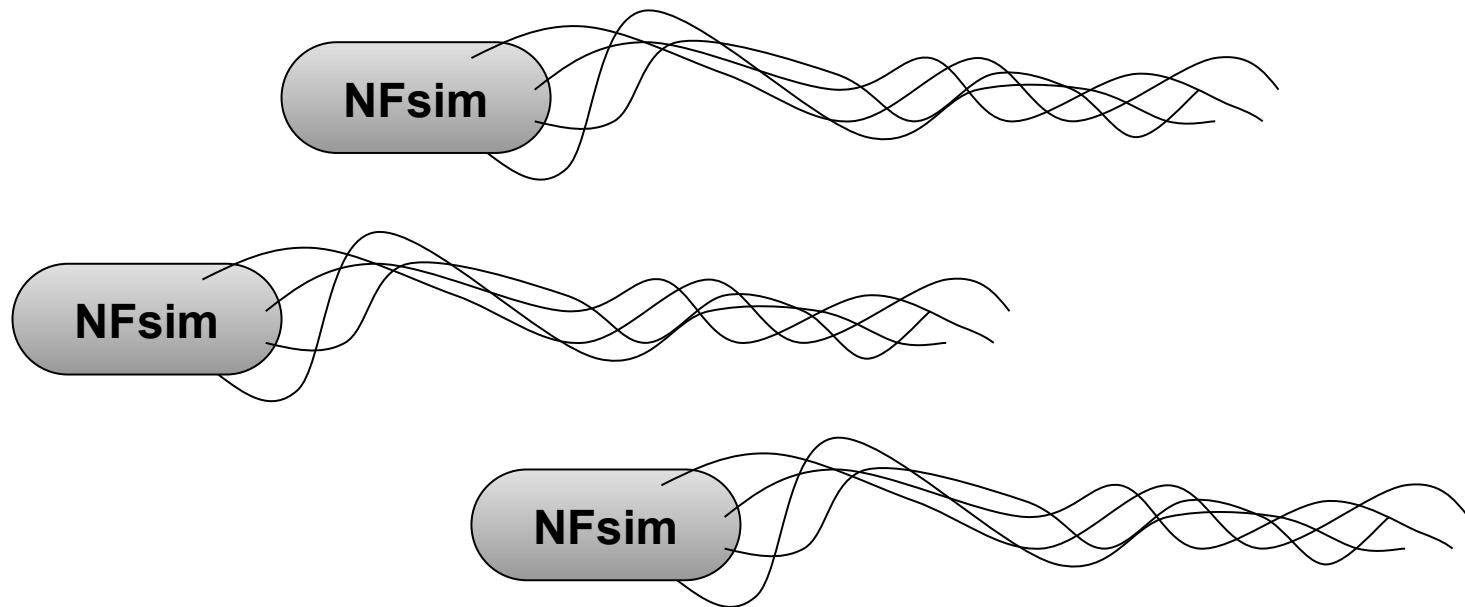
Molecular Level
Interactions



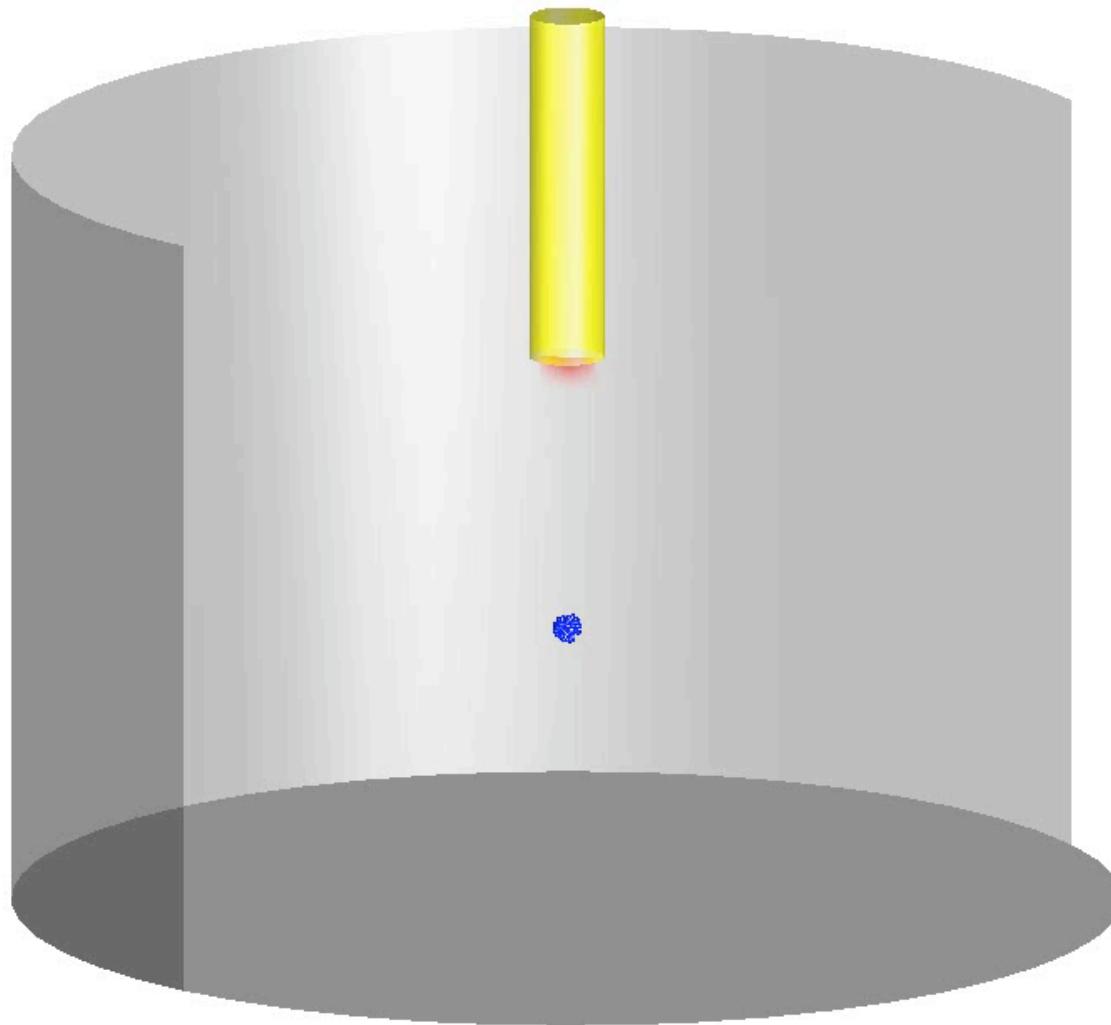
Complexity in Chemotaxis Signaling



NFsim can be embedded into other higher level agents



Digital Chemotaxis Experiments



Elapsed Time: 0 minutes

**200 *E. coli* Cells
2mm from Capillary
10mM Attractant**

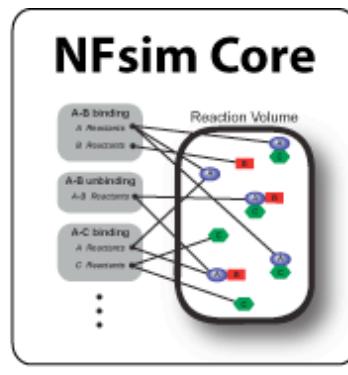
40 min simulation

BioNetGen Interface, Simulation Methods

Faeder Lab
University of Pittsburgh



**Applications to Medicine,
Knowledge Representation**
Gary An Lab
Northwestern University



Emonet Lab
Yale University

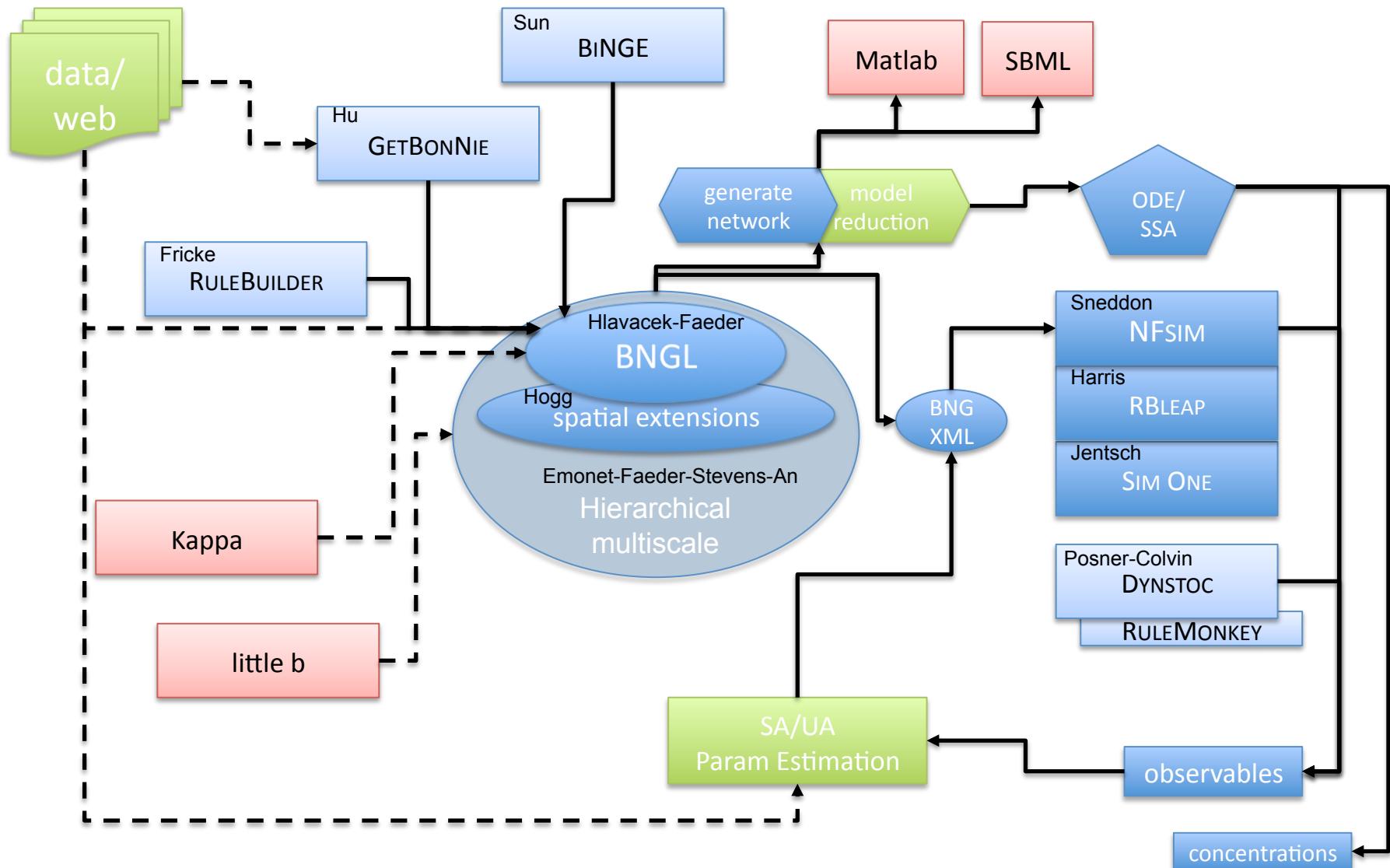
Parallel Computing
Rick Stevens Group
Argonne National Lab



Conclusions

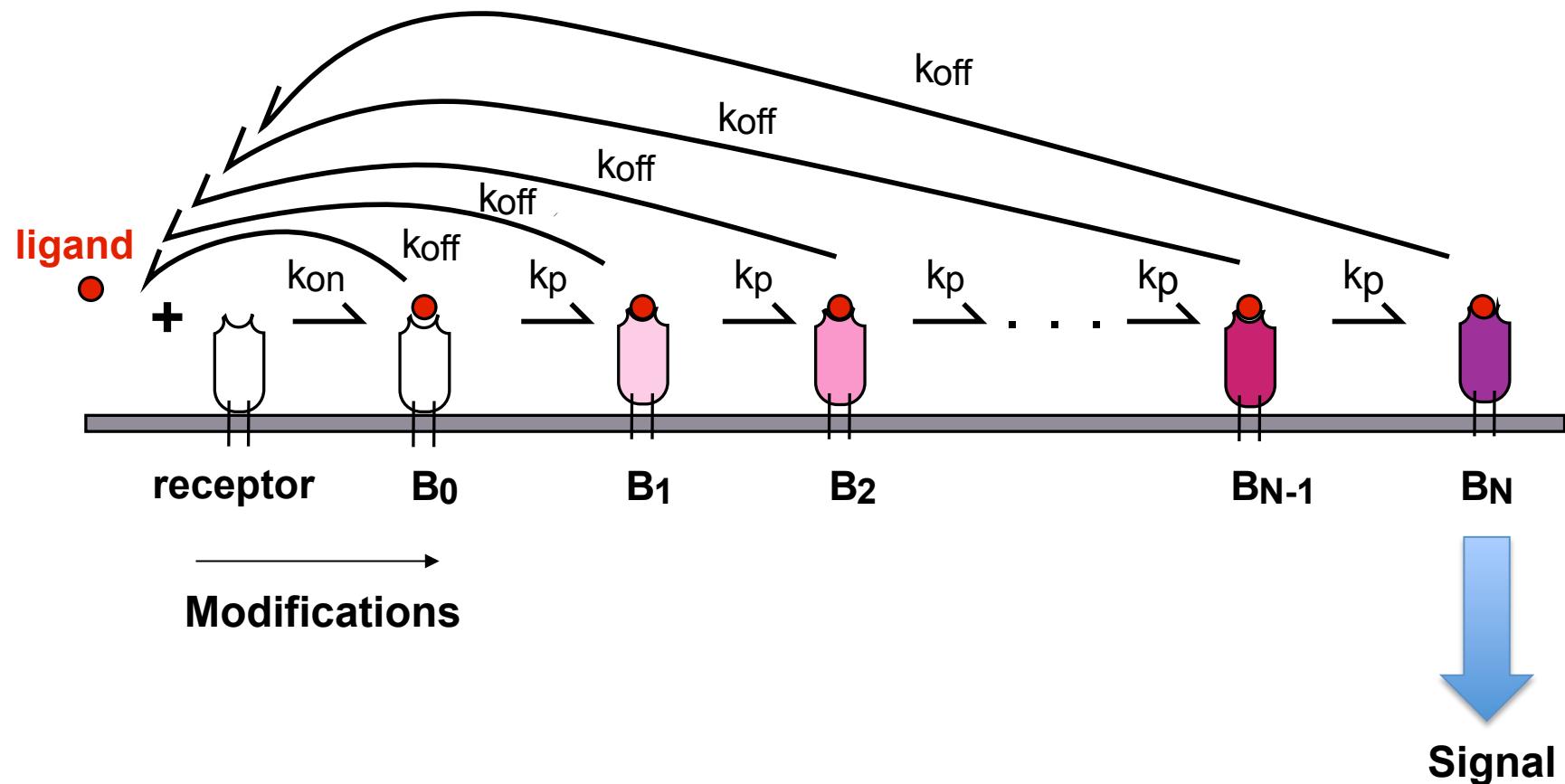
- Kinetics and stoichiometry of complex formation can have a profound effect in signal transduction.
- Modeling these effects requires a new approach to modeling that addresses the issue of *combinatorial complexity*.
- Rule-based (or *interaction-based*) modeling is such an approach.
- *Network-free* simulation is a powerful technique that circumvents combinatorial complexity.
- *Network-free* enables simulation of arbitrarily complex intracellular networks and allows embedding in multi-scale models.

BioNetGen Universe (circa Summer, 2009)



Kinetic Proofreading in Receptor Signaling

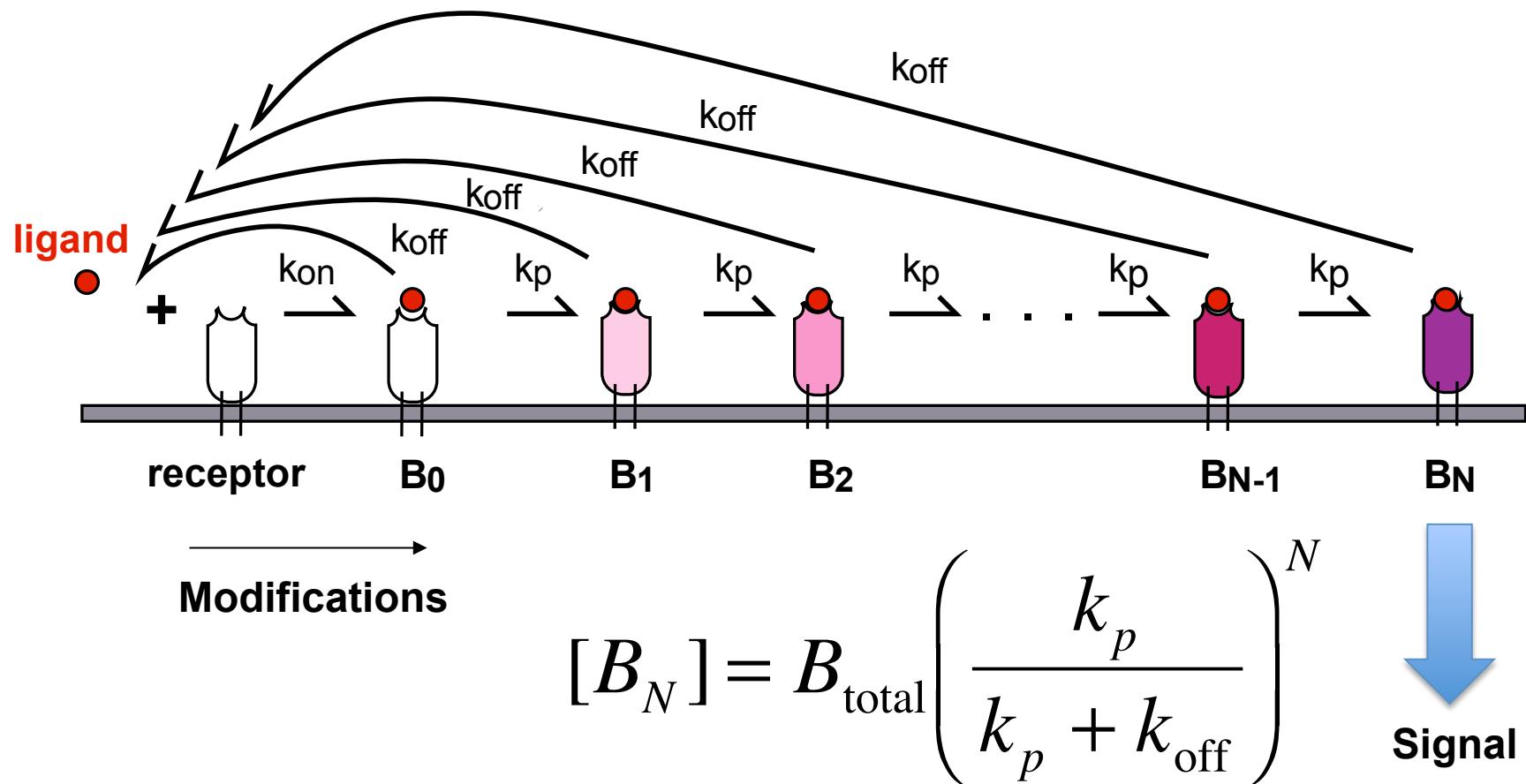
- Ligand dissociation rate can determine ligand efficacy



T. W. McKeithan, PNAS, 92, 5042-5046 (1995).

Kinetic Proofreading in Receptor Signaling

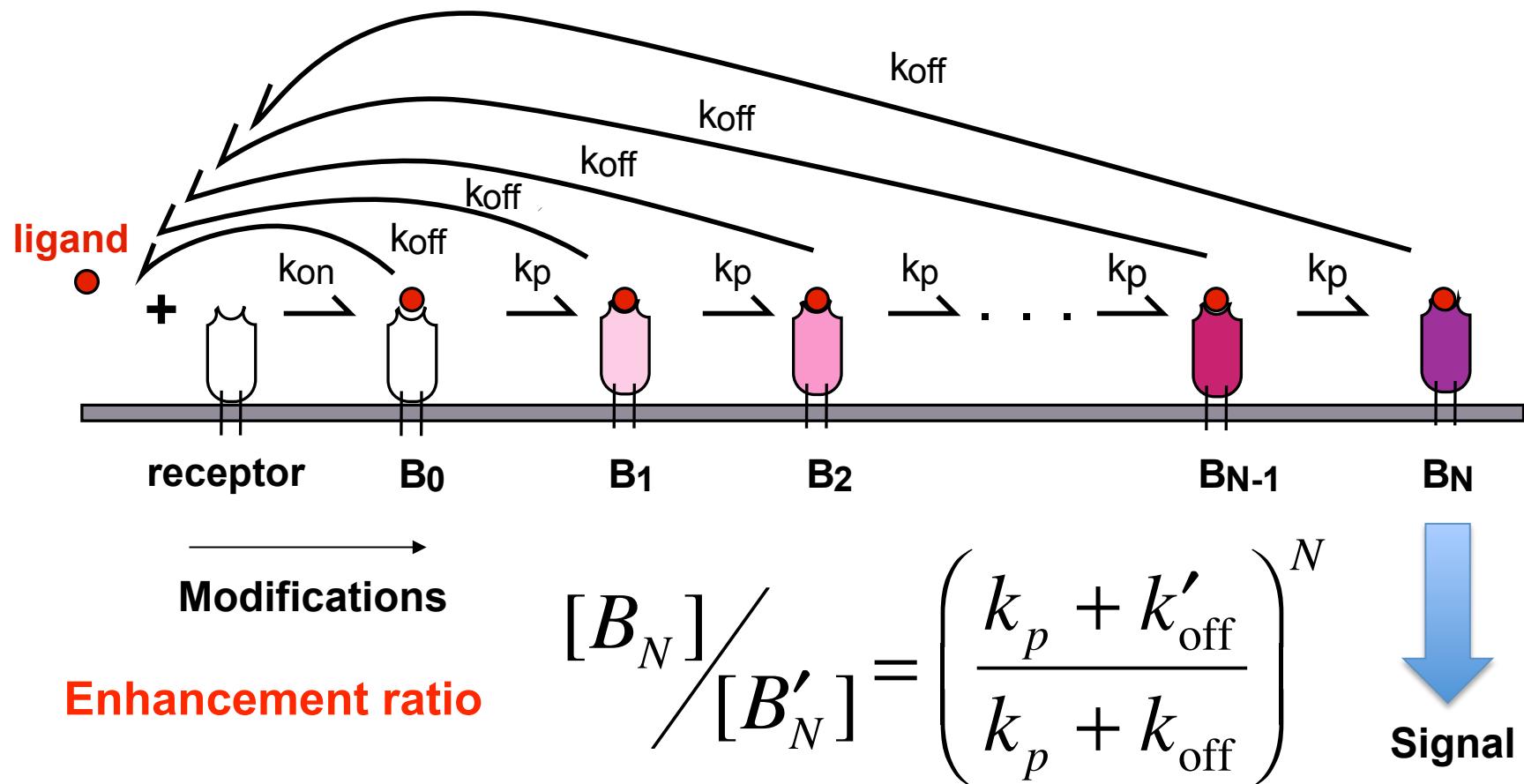
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T. W. McKeithan, PNAS, **92**, 5042-5046 (1995).

Kinetic Proofreading in Receptor Signaling

- Ligand dissociation rate can determine ligand efficacy

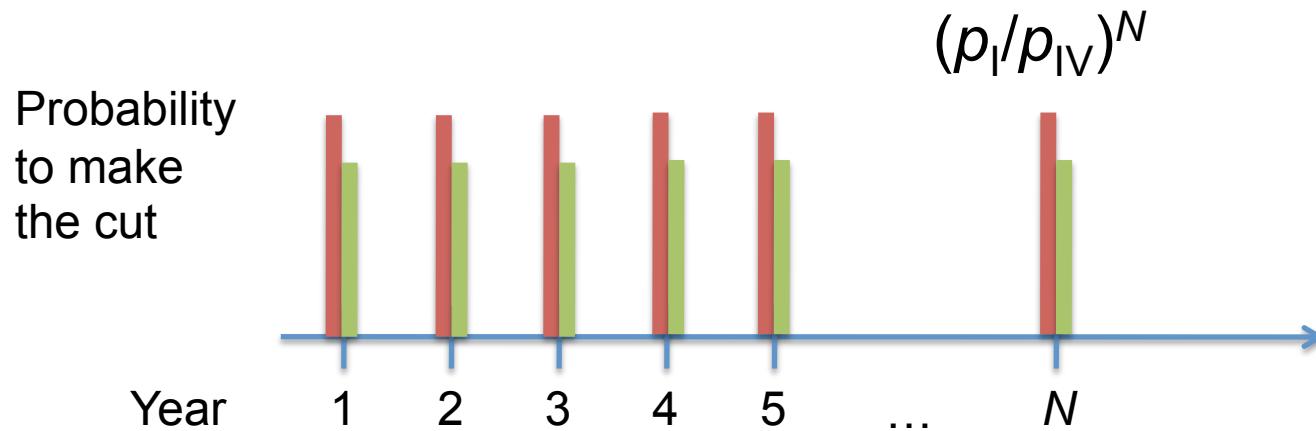


T. W. McKeithan, PNAS, 92, 5042-5046 (1995).

Kinetic Proofreading in Sports

Malcolm Gladwell, *Outliers*.

- Many sports (and education systems) have cutoff dates to establish eligibility
- Having a birthdate close to the cutoff date confers a small but tangible advantage



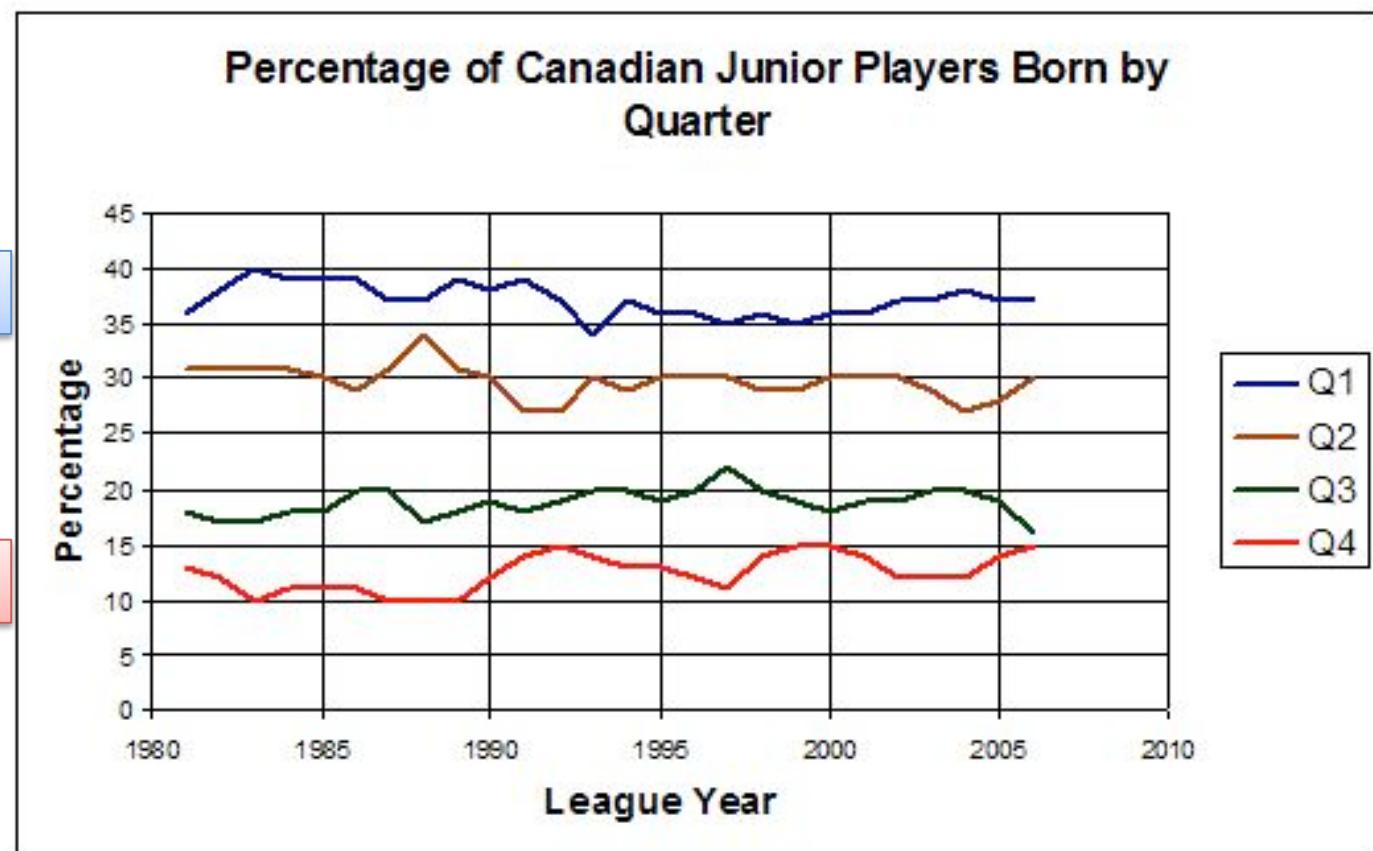
Kinetic Proofreading in Sports

Malcolm Gladwell, *Outliers*.

Born Jan-Mar

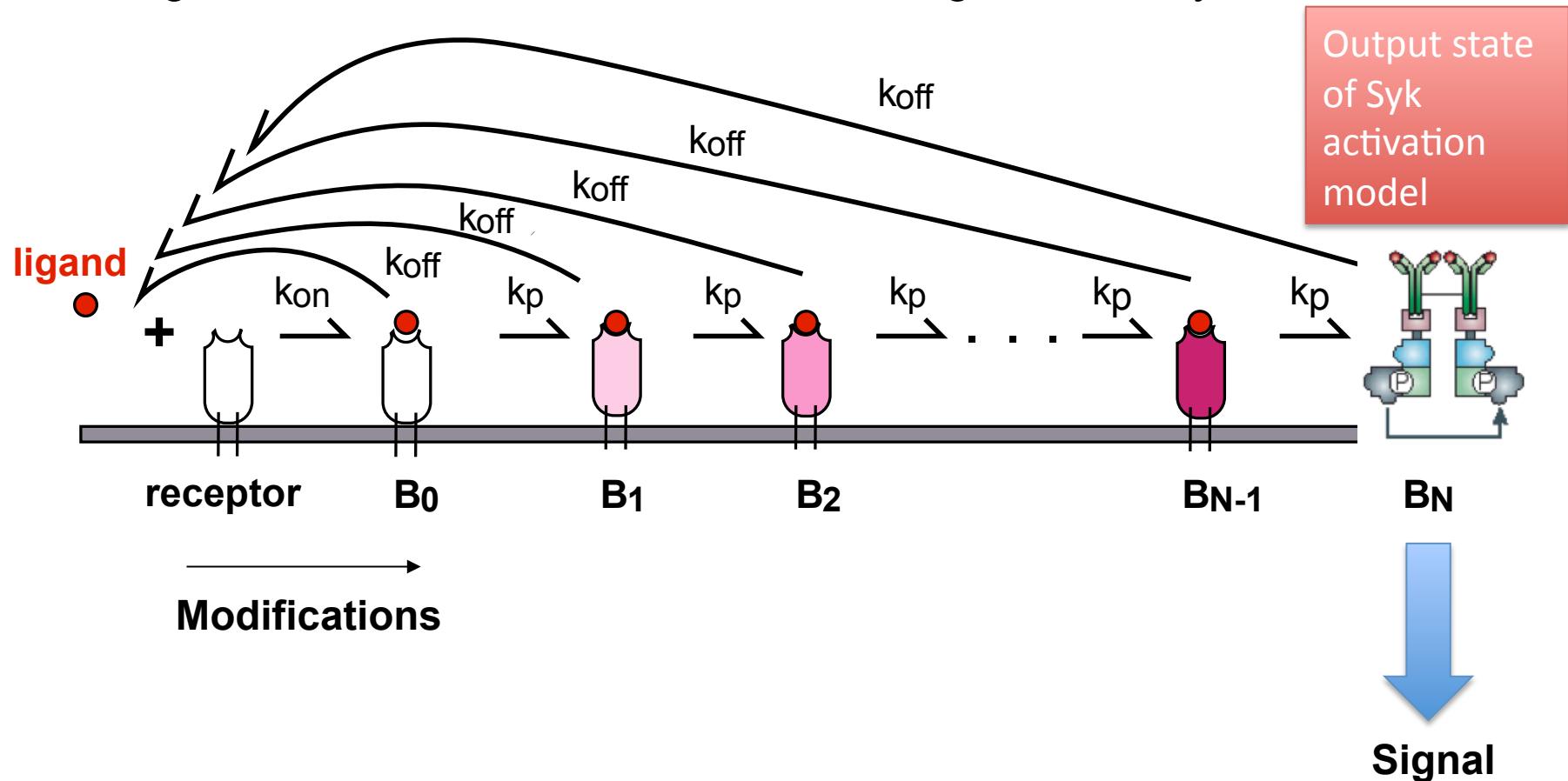
2.5-4 fold!

Born Oct-Dec



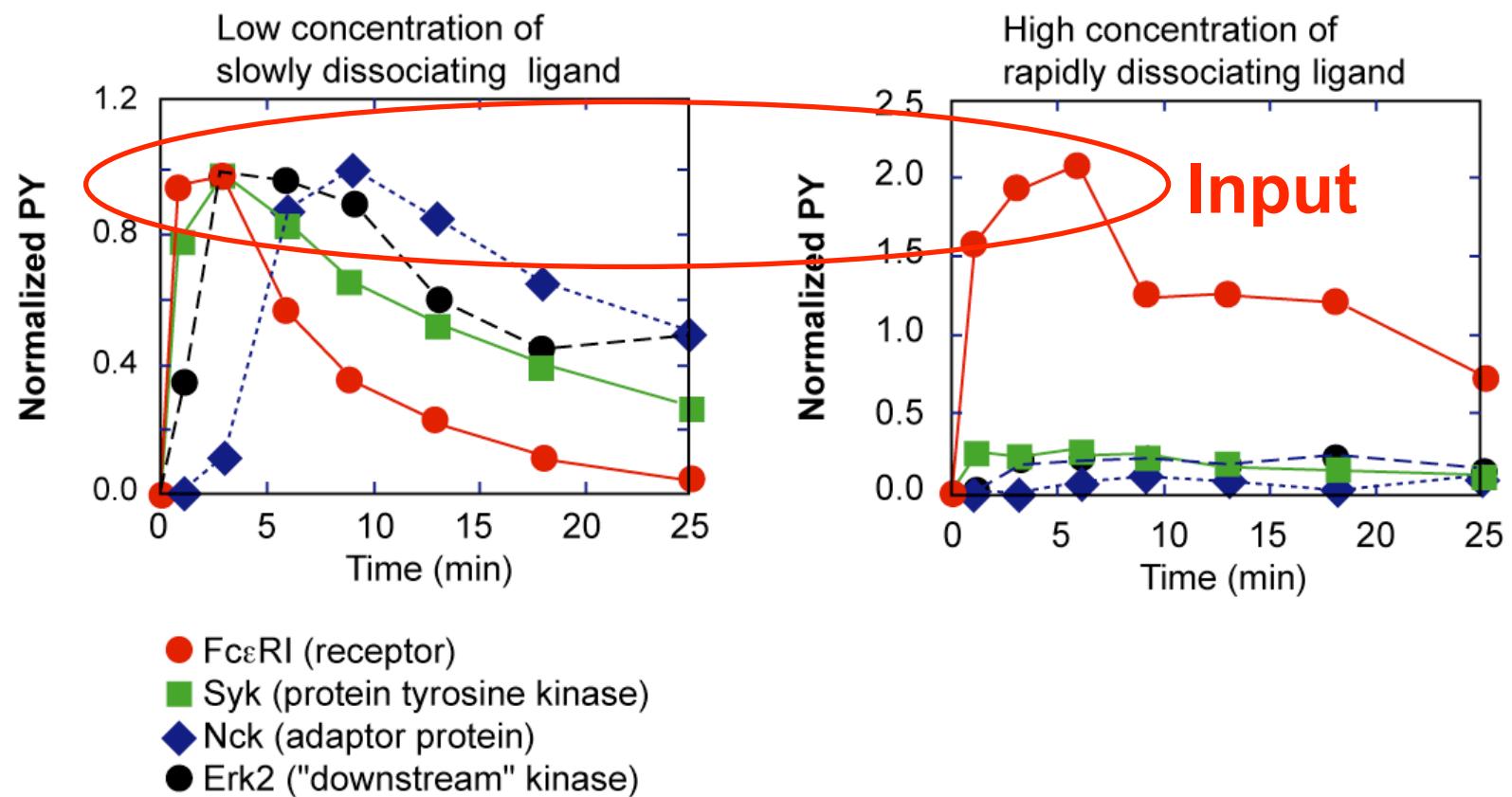
Kinetic Proofreading in Receptor Signaling

- Ligand dissociation rate can determine ligand efficacy



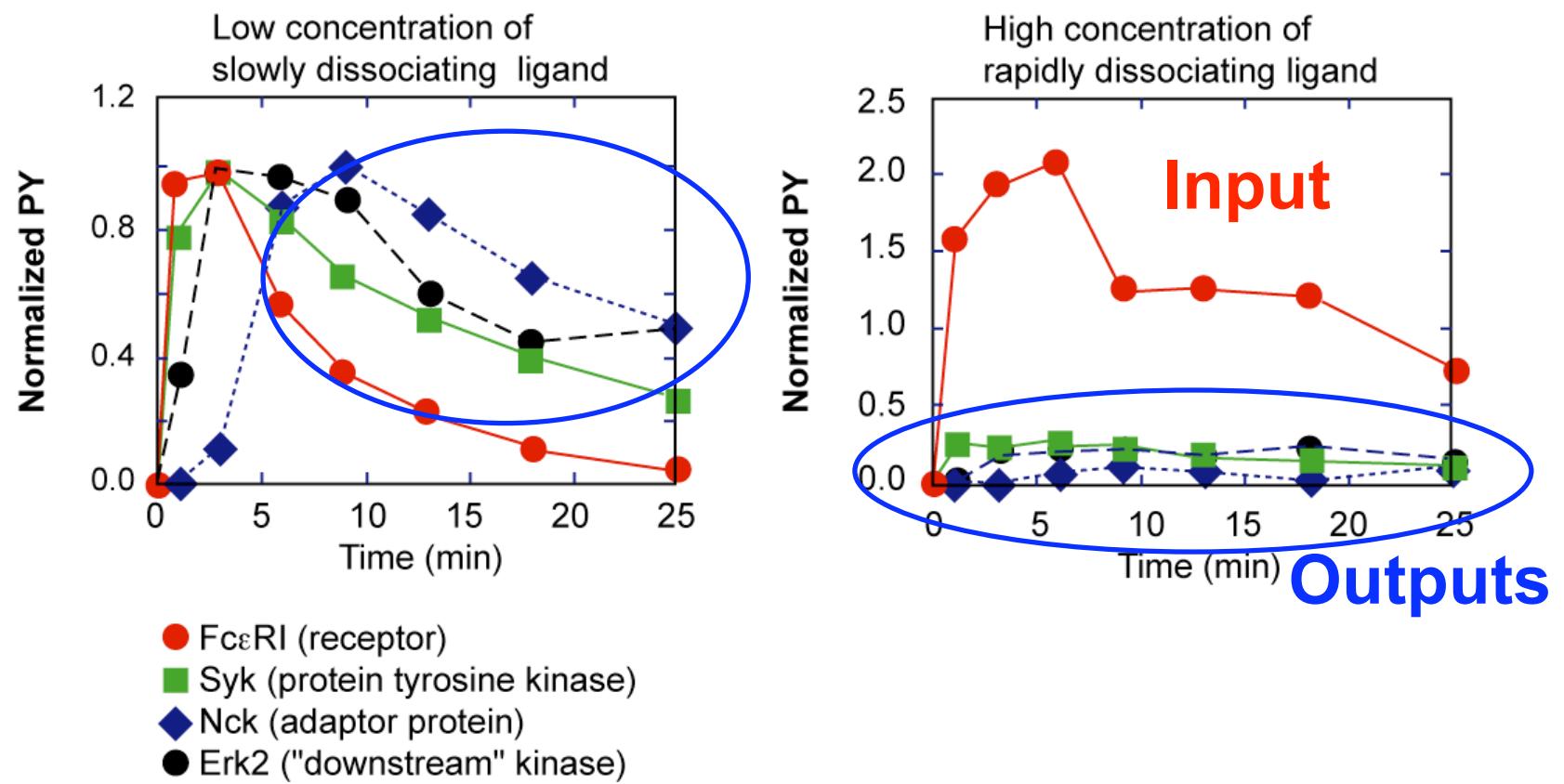
T. W. McKeithan, PNAS, 92, 5042-5046 (1995).

Evidence for Kinetic Proofreading in Mast Cell Responses to Two Ligands



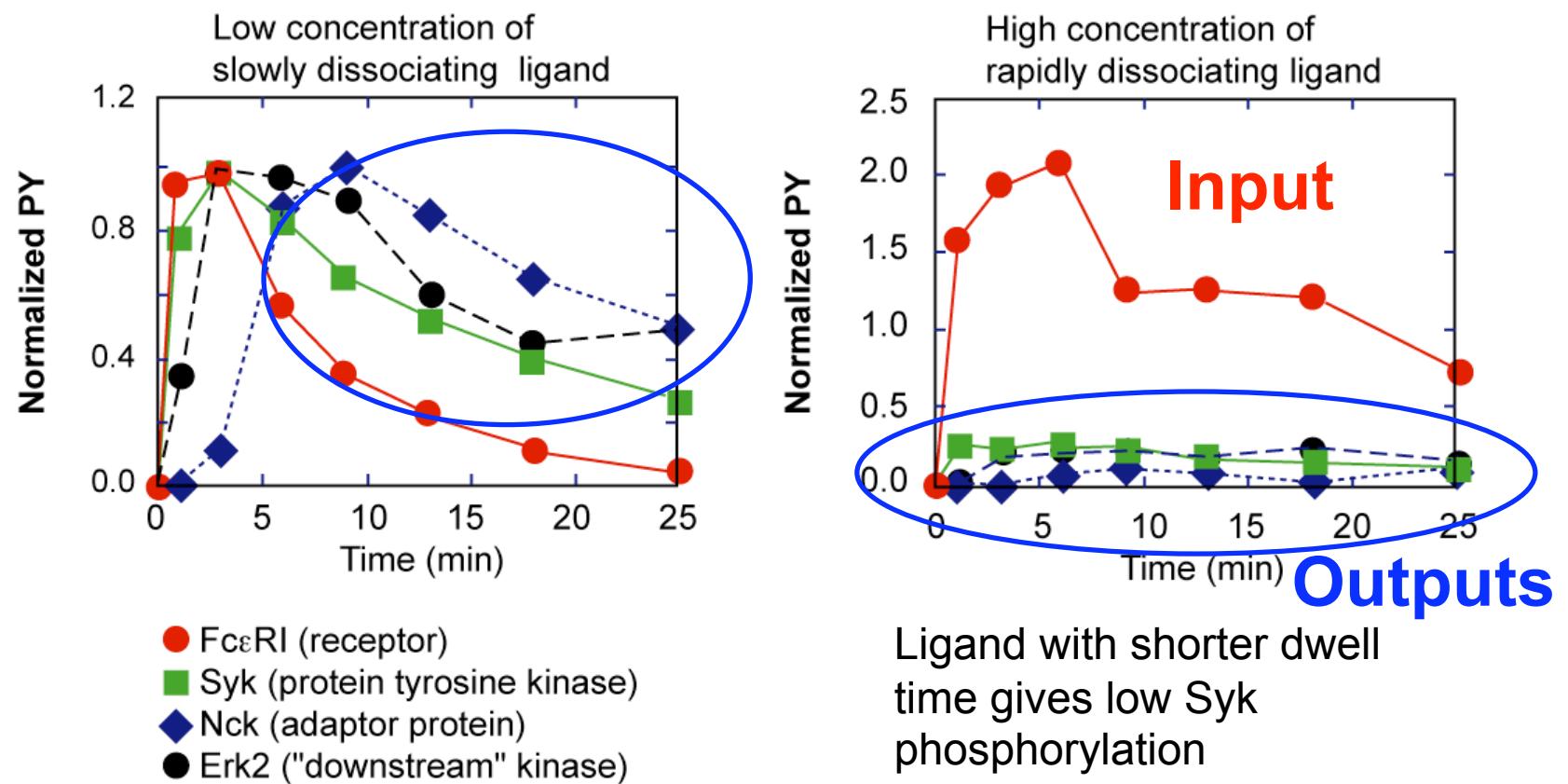
Torigoe, Inman & Metzger, *Science*, **281**, 568 (1998)

Evidence for Kinetic Proofreading in Mast Cell Responses to Two Ligands



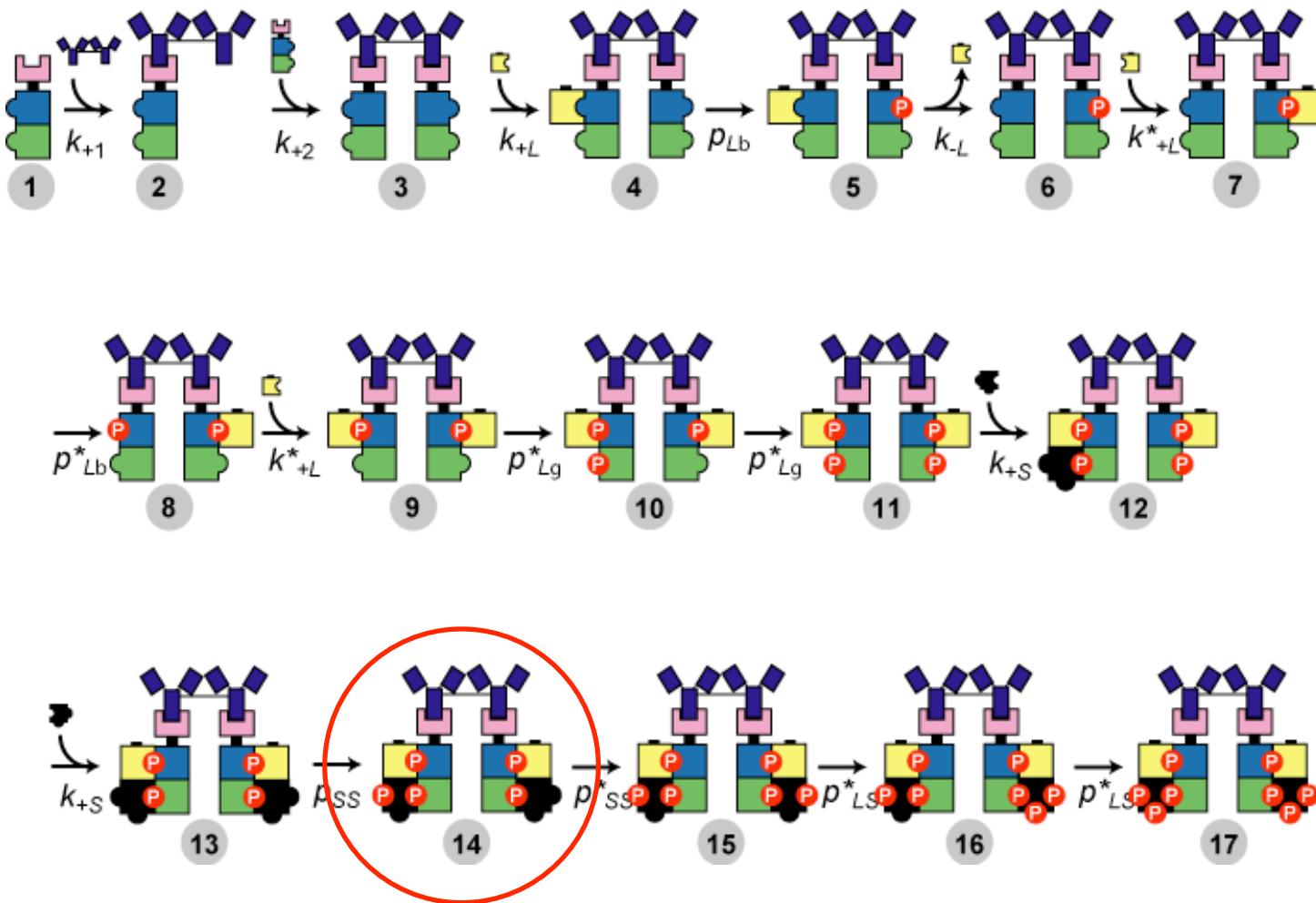
Torigoe, Inman & Metzger, *Science*, **281**, 568 (1998)

Evidence for Kinetic Proofreading in Mast Cell Responses to Two Ligands

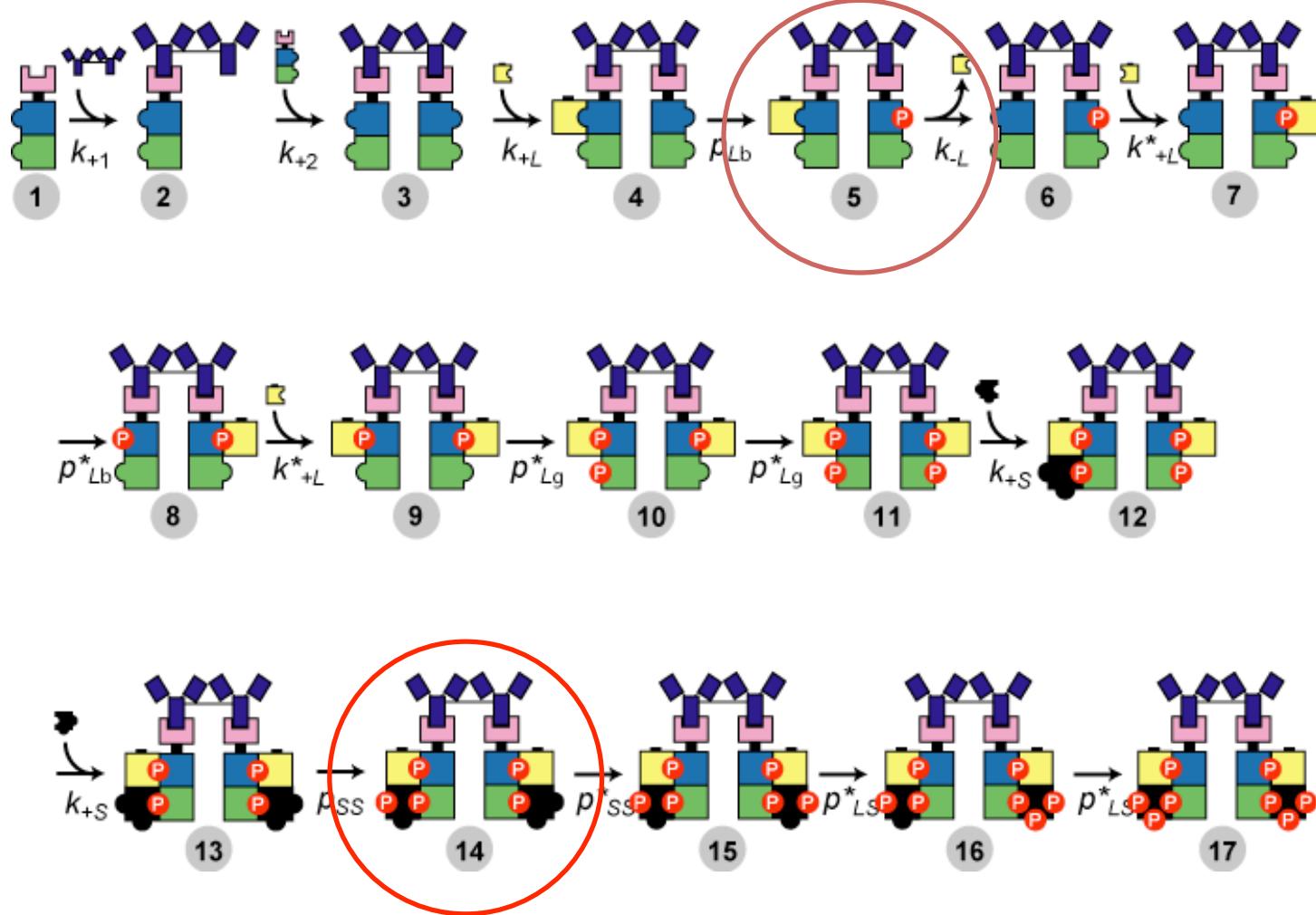


Torigoe, Inman & Metzger, *Science*, 281, 568 (1998)

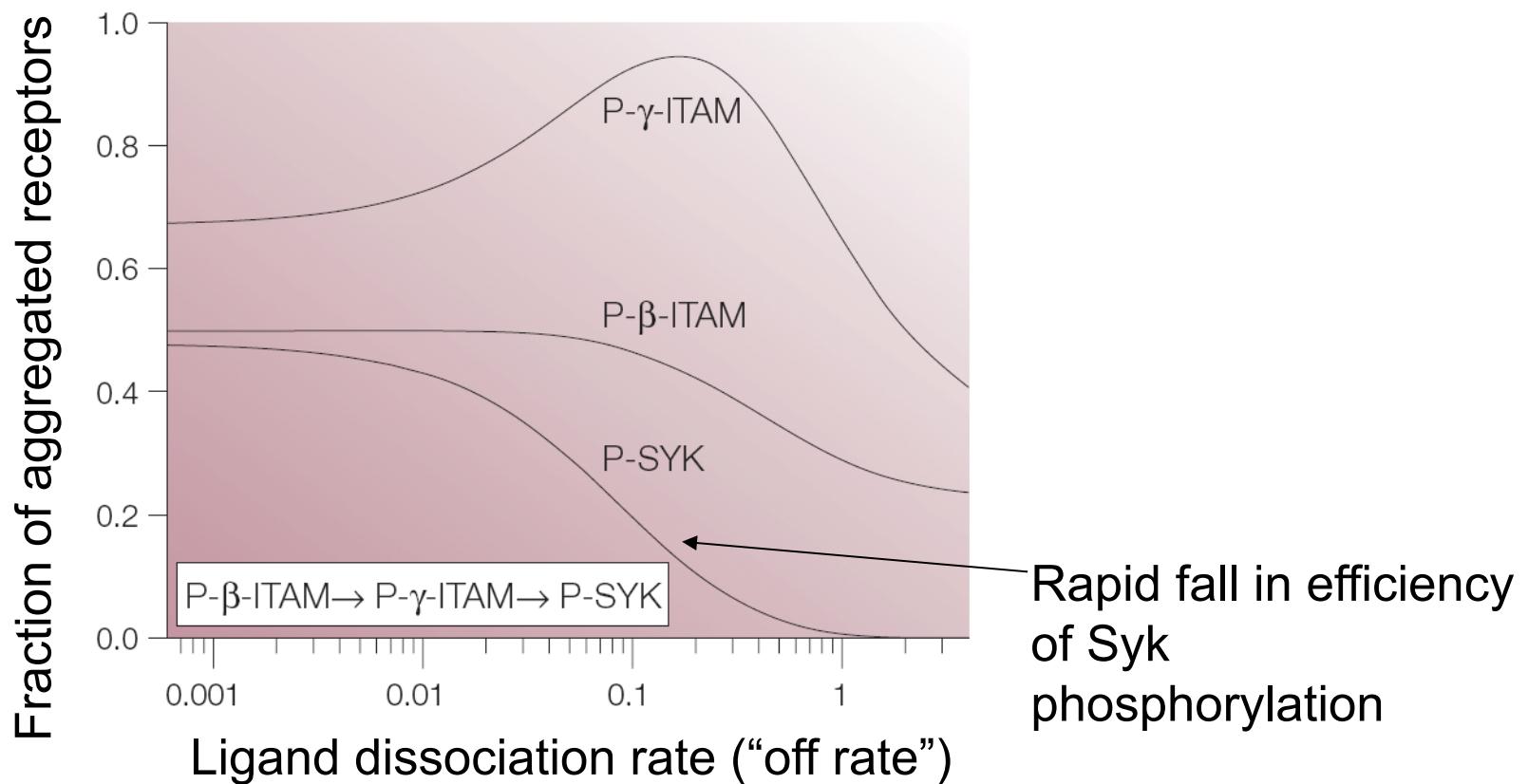
Large number of reaction events required for Syk activation



Small number of reaction events required for receptor phosphorylation



Kinetic proofreading of Syk activation but not receptor phosphorylation



Goldstein *et al.* (2004) *Nat. Rev. Immunol.* **4**, 445-456.

Bimodal dose-response curves

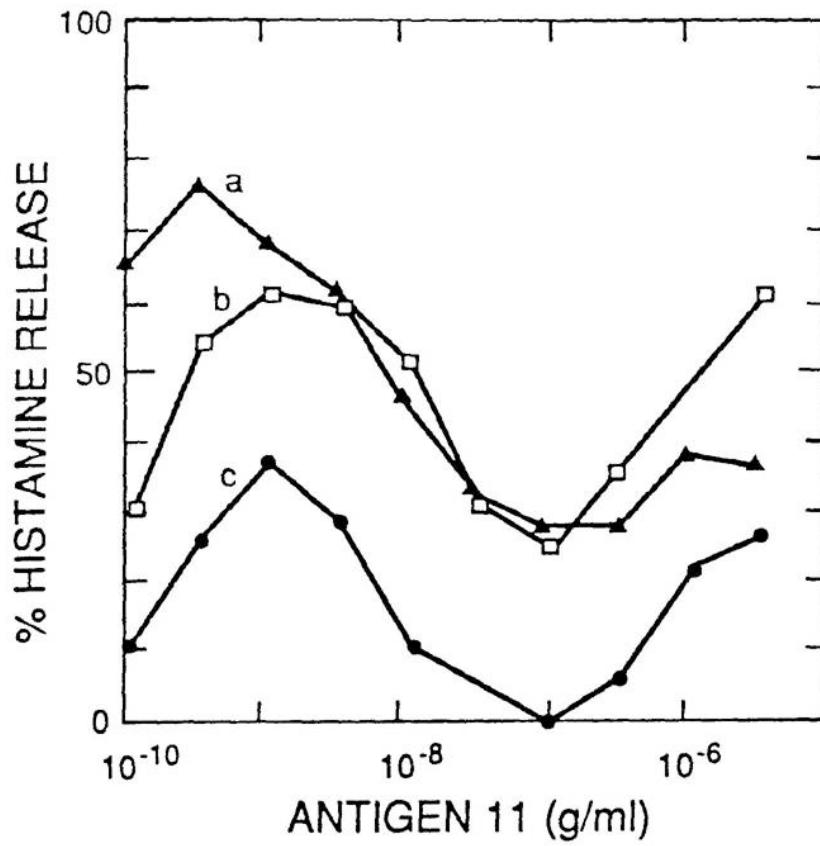


FIGURE 12 Histamine release curves from basophils from three donors who were allergic to the dust mite *Dermatophagoides farinae* (Weyer et al., 1982). Antigen 11 is a highly purified component of *D. farinae*. DeLisi and Siraganian (1979) observed similar double-humped histamine release curves with rabbit basophils.

B. Goldstein, in *Theoretical Immunology, Part One*, Ed. A. S. Perelson

Bimodal dose-response curves

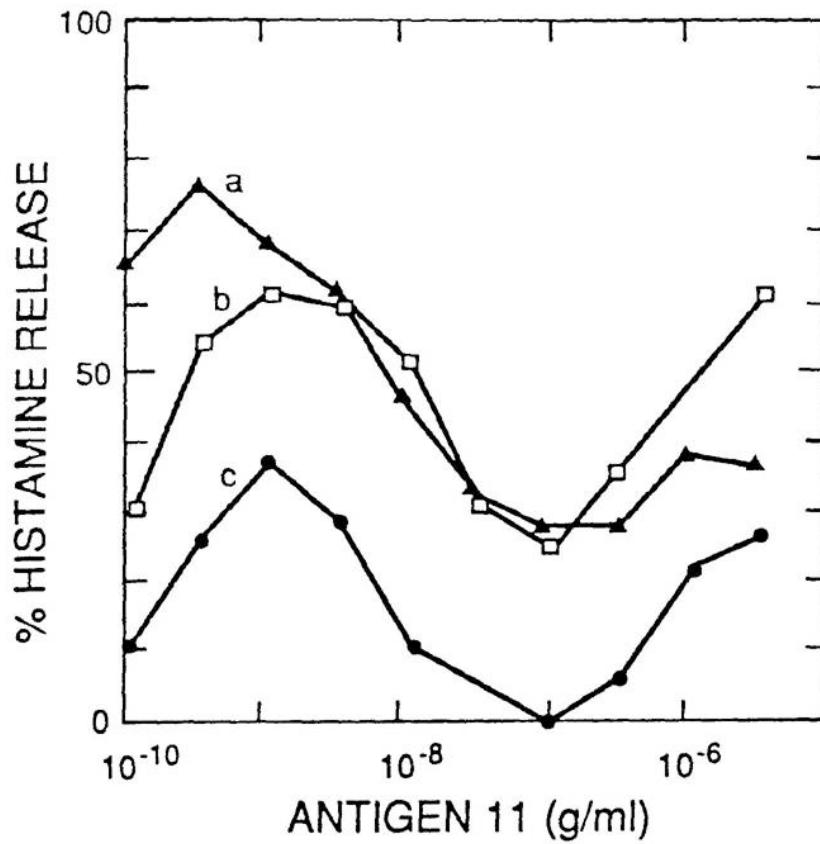


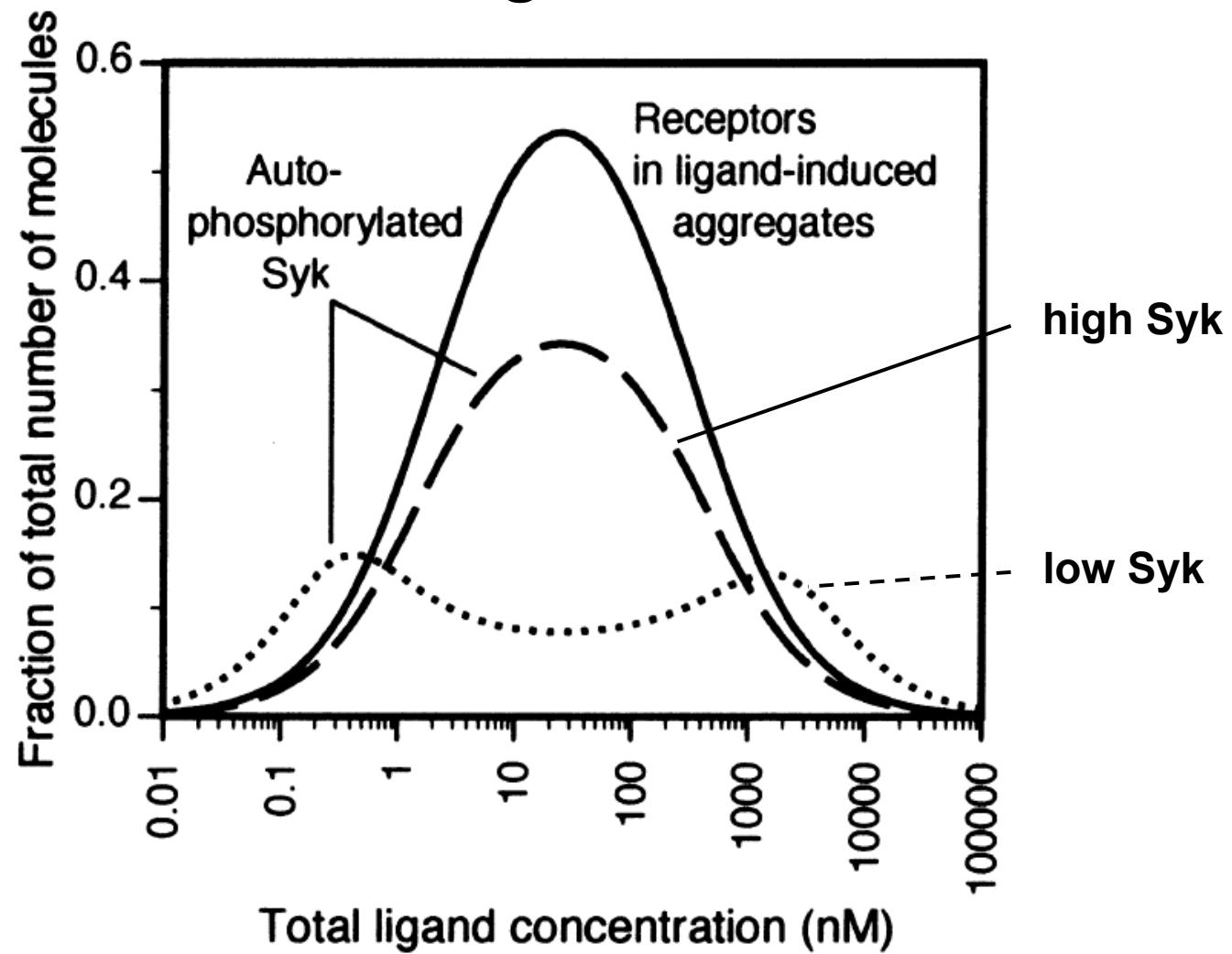
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Syk expression is highly variable in human basophils (5,000-60,000 copies per cell)

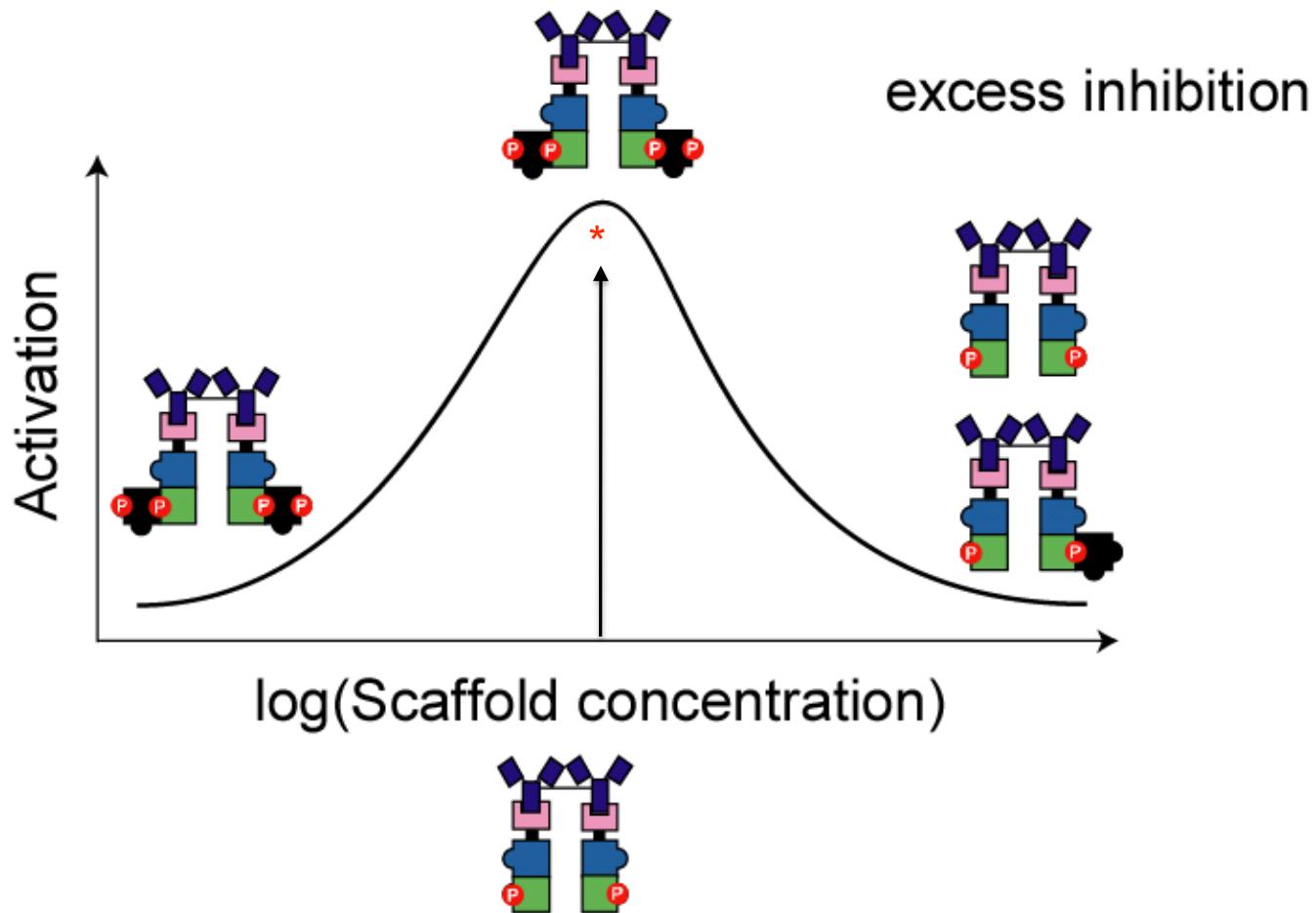
MacGlashan (2007)

B. Goldstein, in *Theoretical Immunology, Part One*, Ed. A. S. Perelson

Dose-response curves for reversibly binding ligand

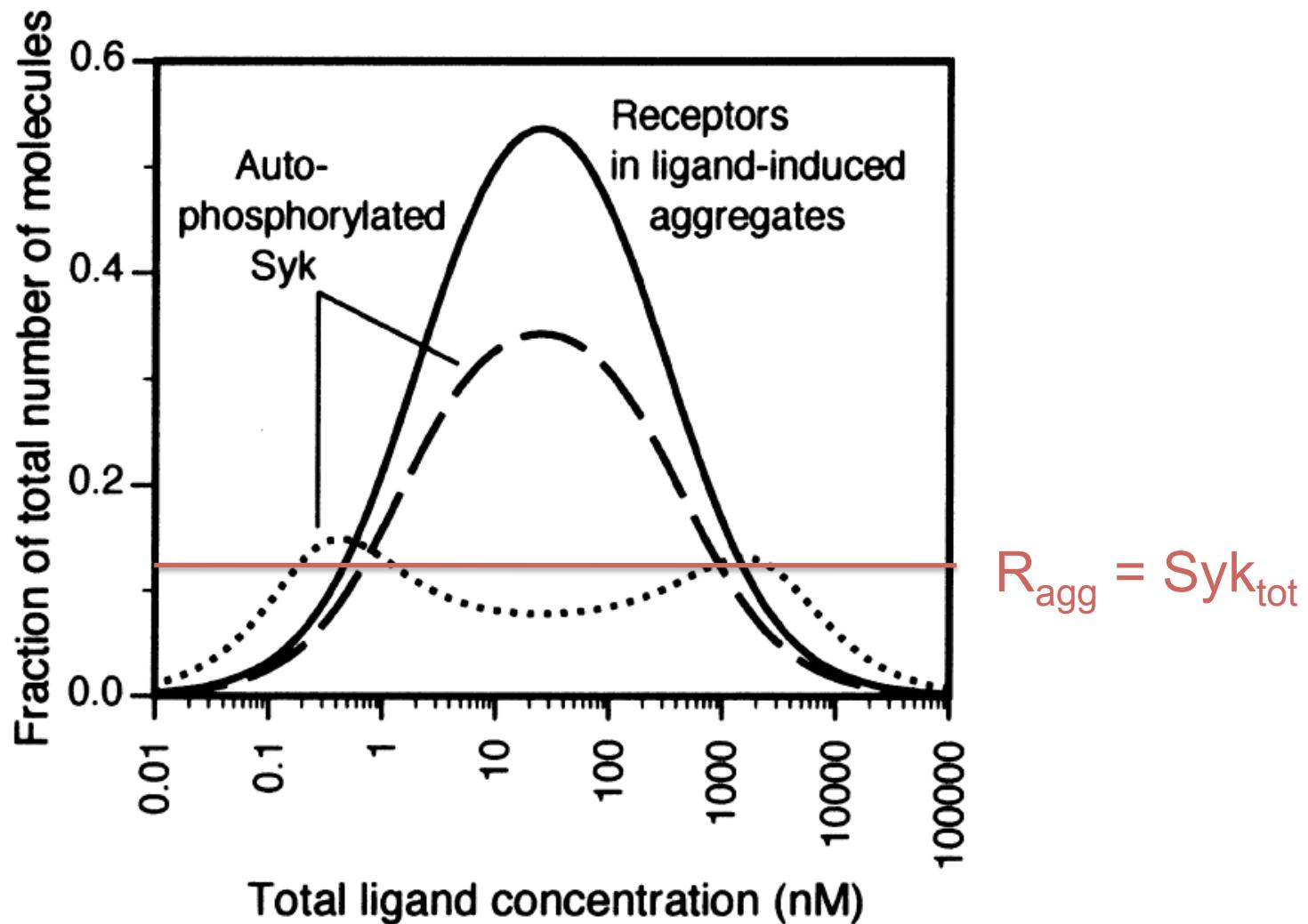


The multivalent scaffold effect



*Syk and scaffold concentrations are equal

Bimodal response occurs when Syk concentration below maximal number of aggregated receptors



Growth Hormone / Jak2 Signaling

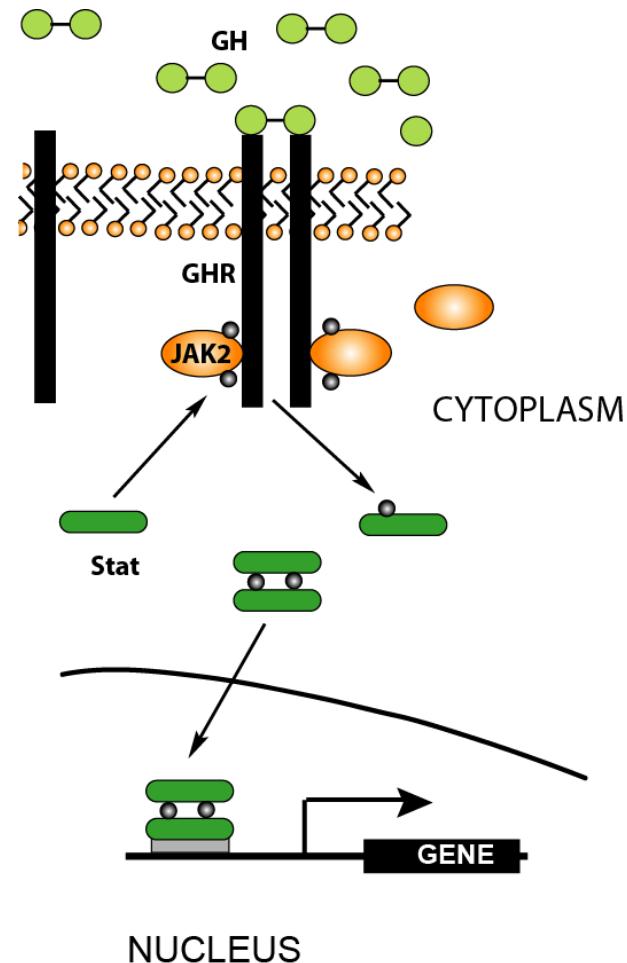


Dipak Barua



Jason Haugh

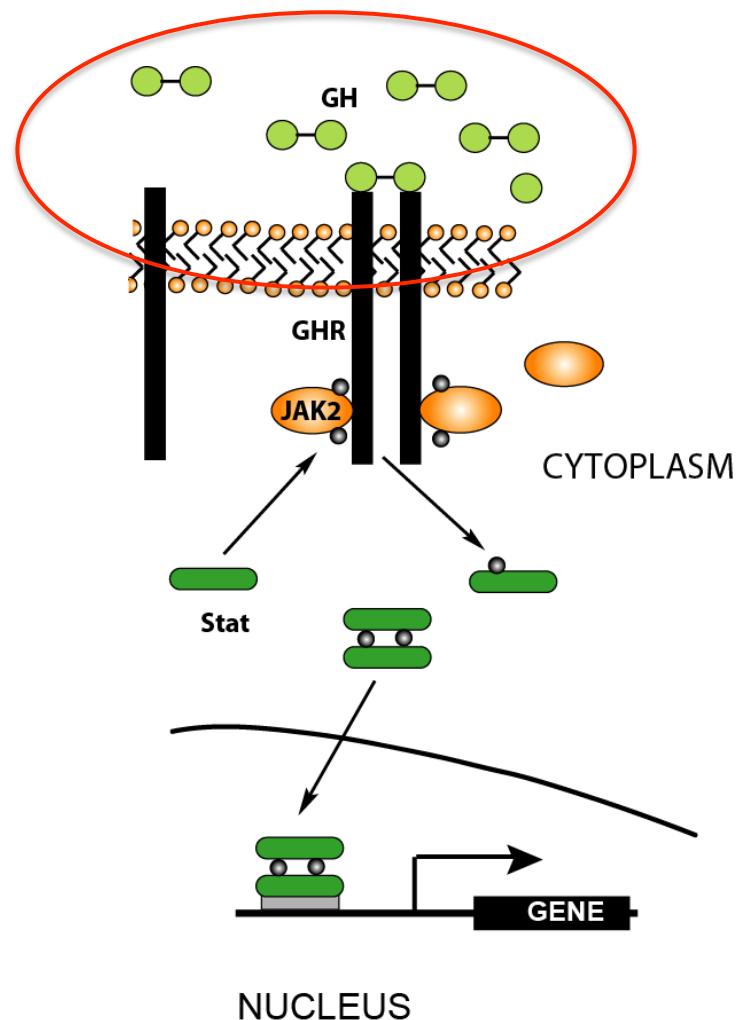
NC State University



Barua et al., *PLoS Comp. Biol.* (2009)

Growth Hormone / Jak2 Signaling

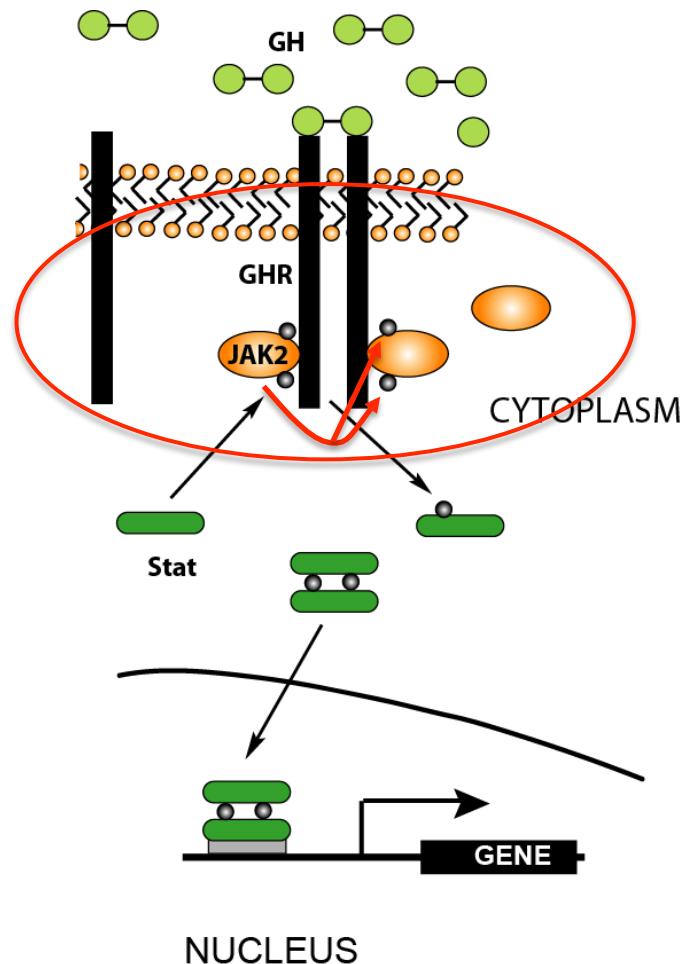
Ligand binding and receptor aggregation



Growth Hormone / Jak2 Signaling

Ligand binding and receptor aggregation

Jak2 recruitment and transautophosphorylation

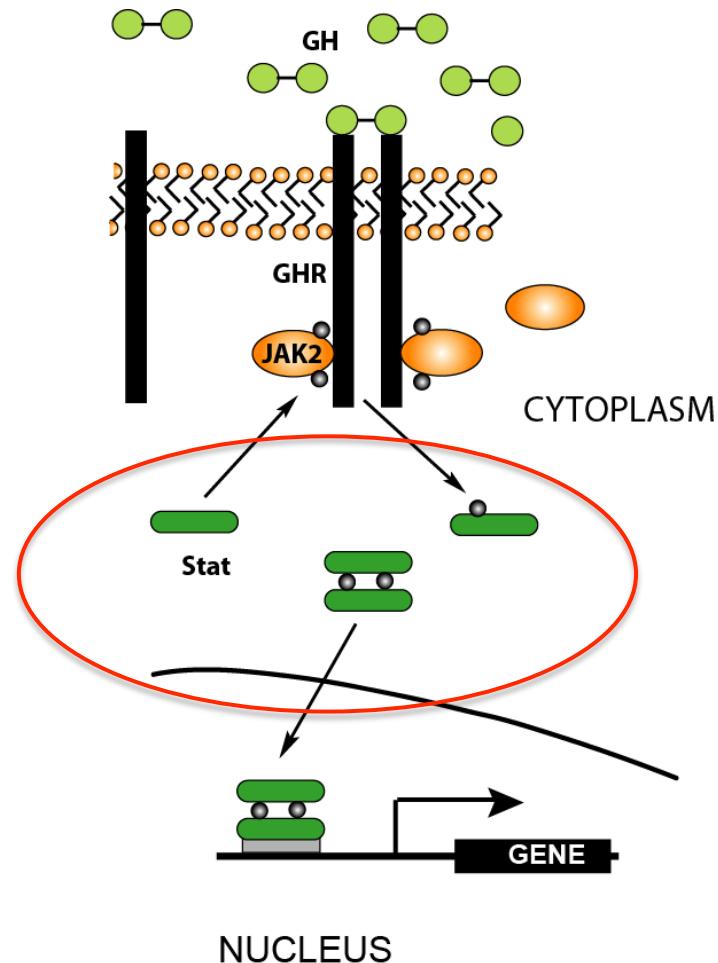


Growth Hormone / Jak2 Signaling

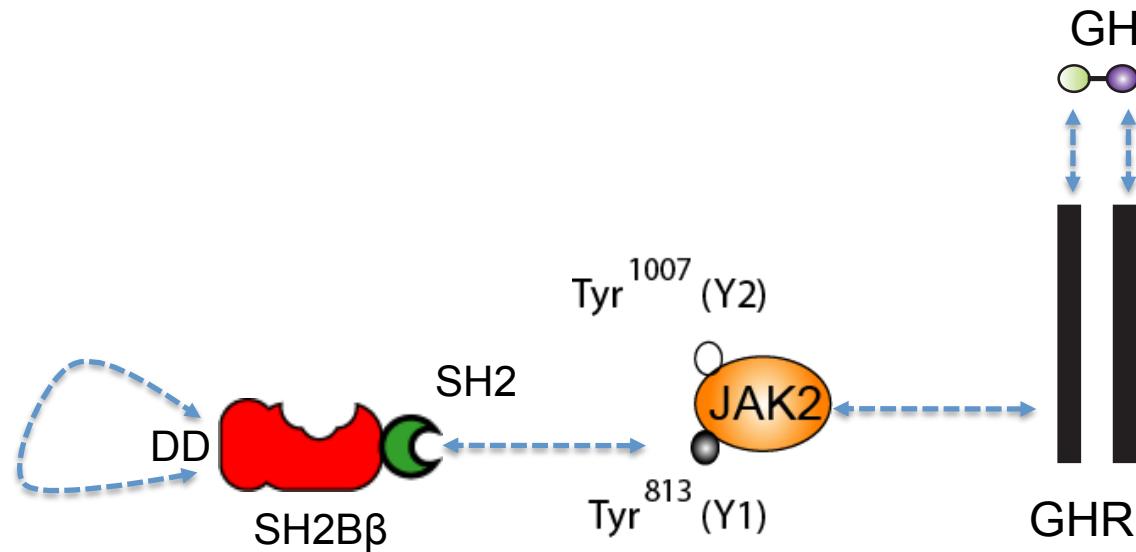
Ligand binding and receptor aggregation

Jak2 recruitment and transautophosphorylation

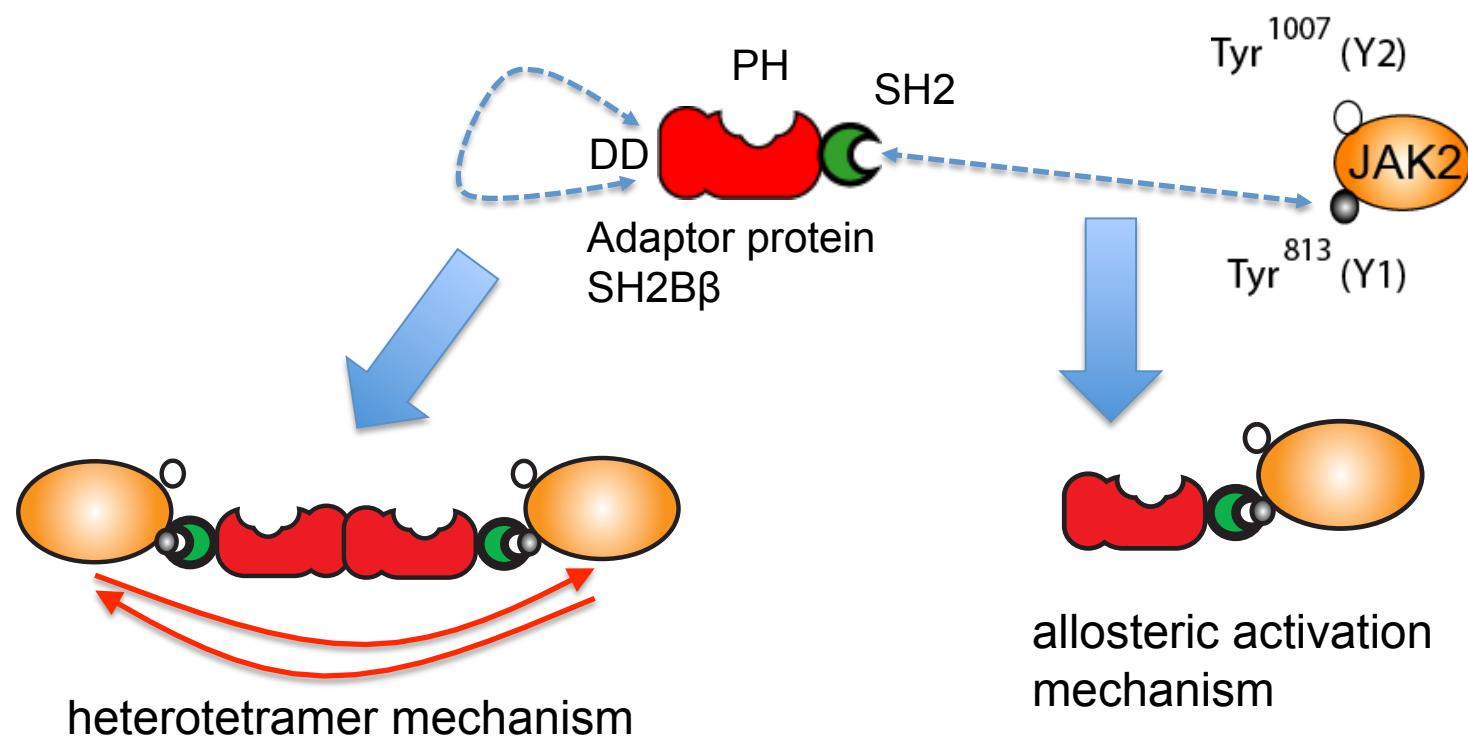
Stat3/5b phosphorylation, dimerization, and translocation



Model of GHR-Jak2-SH2-B β Interactions



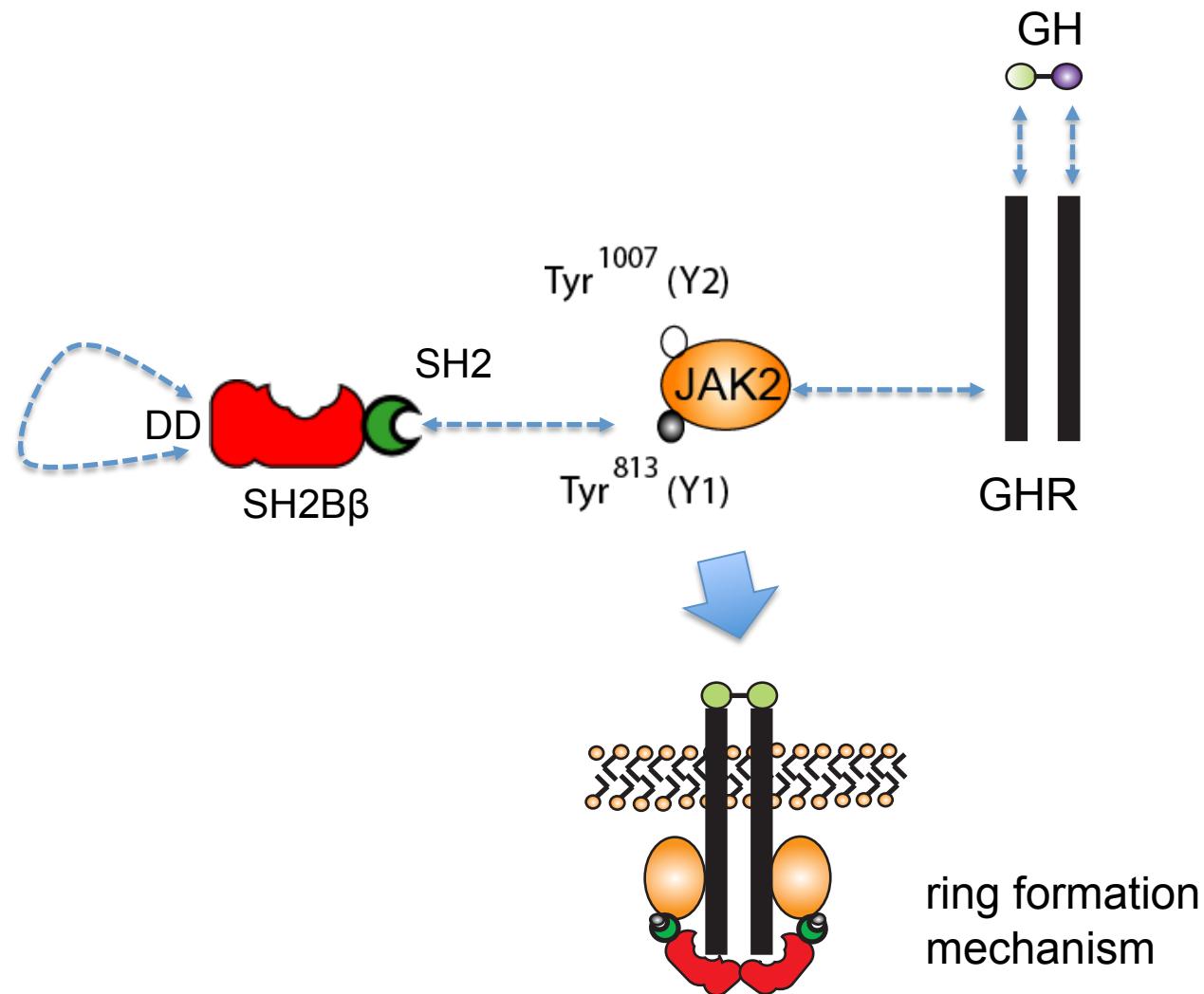
SH2-B β : An Adaptor that Modulates Jak2 Activation by GHR



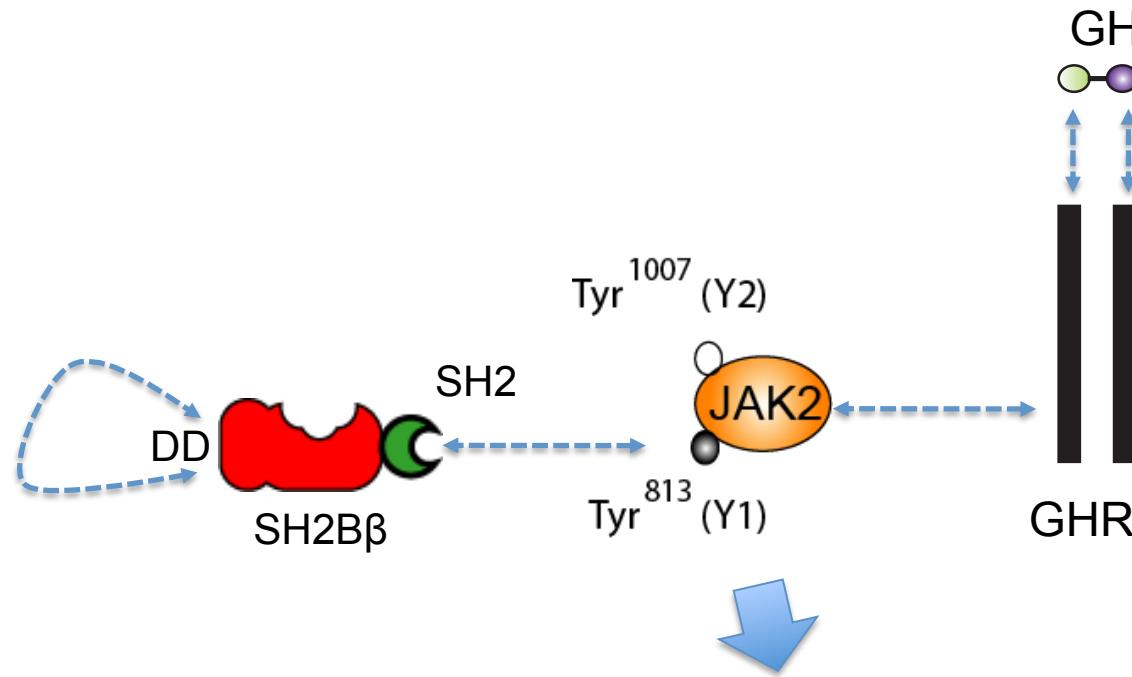
Nishi et al. (2005) *Mol Cell Biol* **25**,
2607-2621.

Rui et al. (2000) *Mol Cell Biol*
20, 3168-3177

Model of GHR-Jak2-SH2-B β Interactions - Third Mechanism

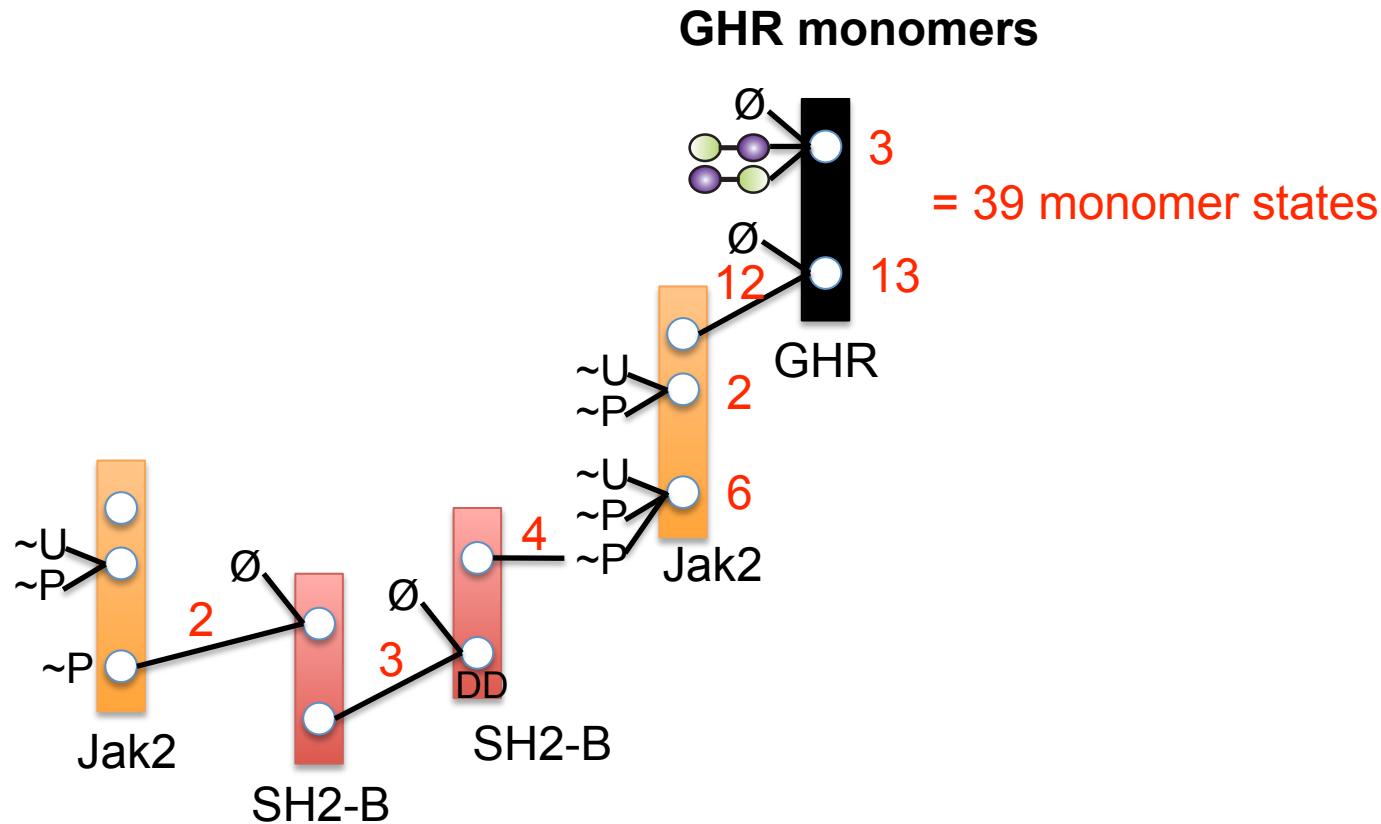


Model of GHR-Jak2-SH2-B β Interactions - Complexity

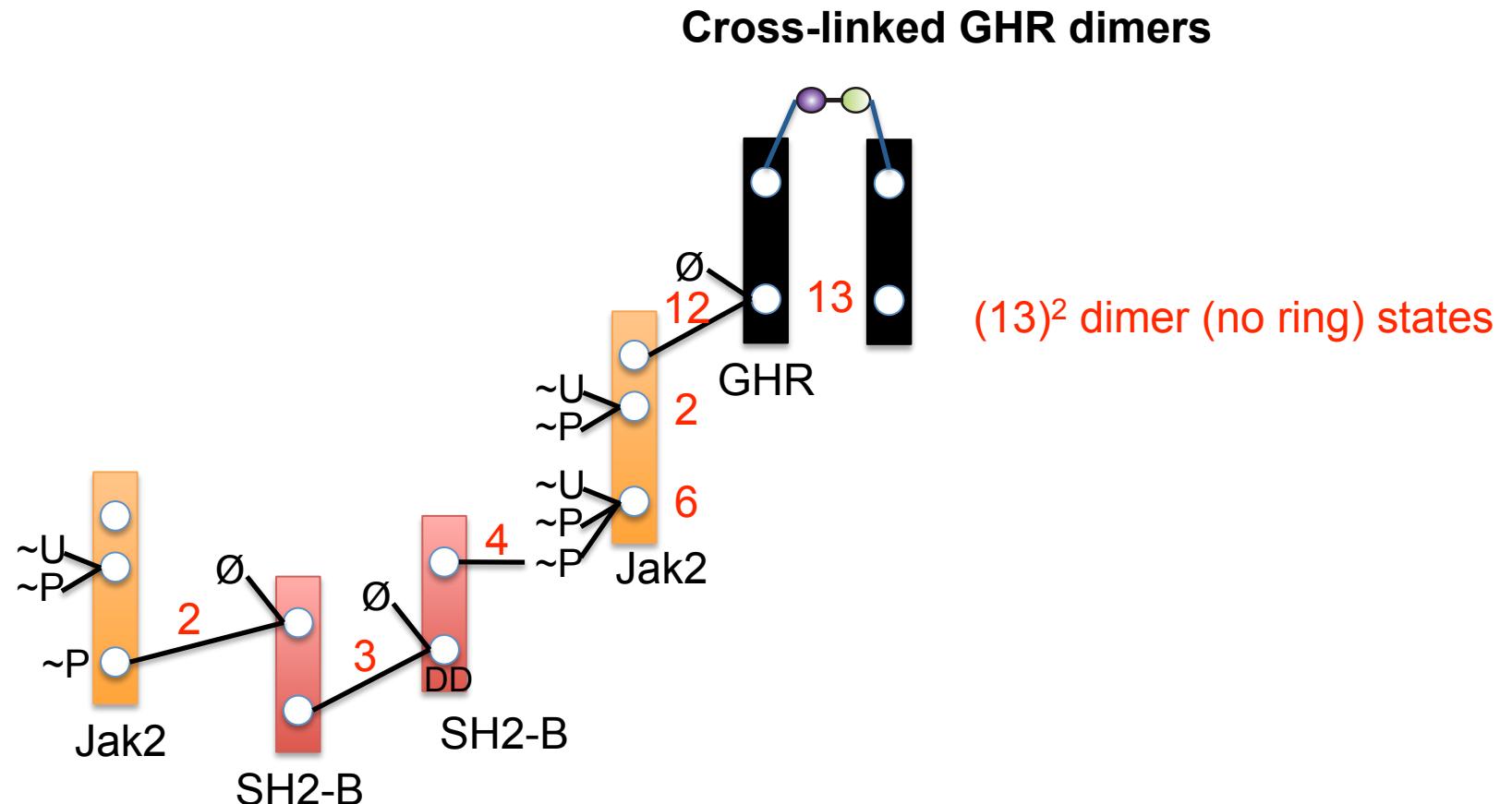


**741 Species / 5,033
Reactions**

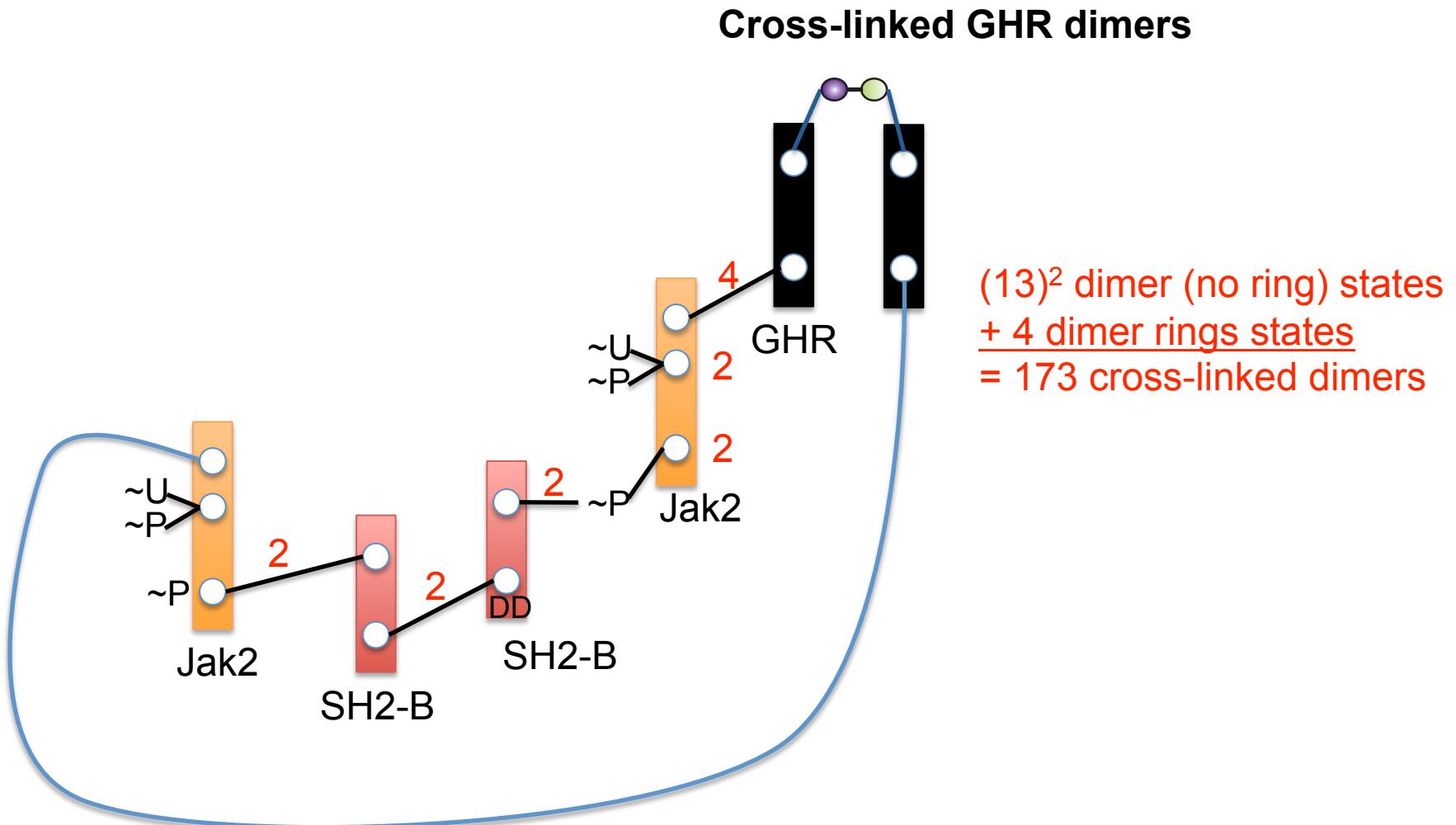
Combinatorial Complexity



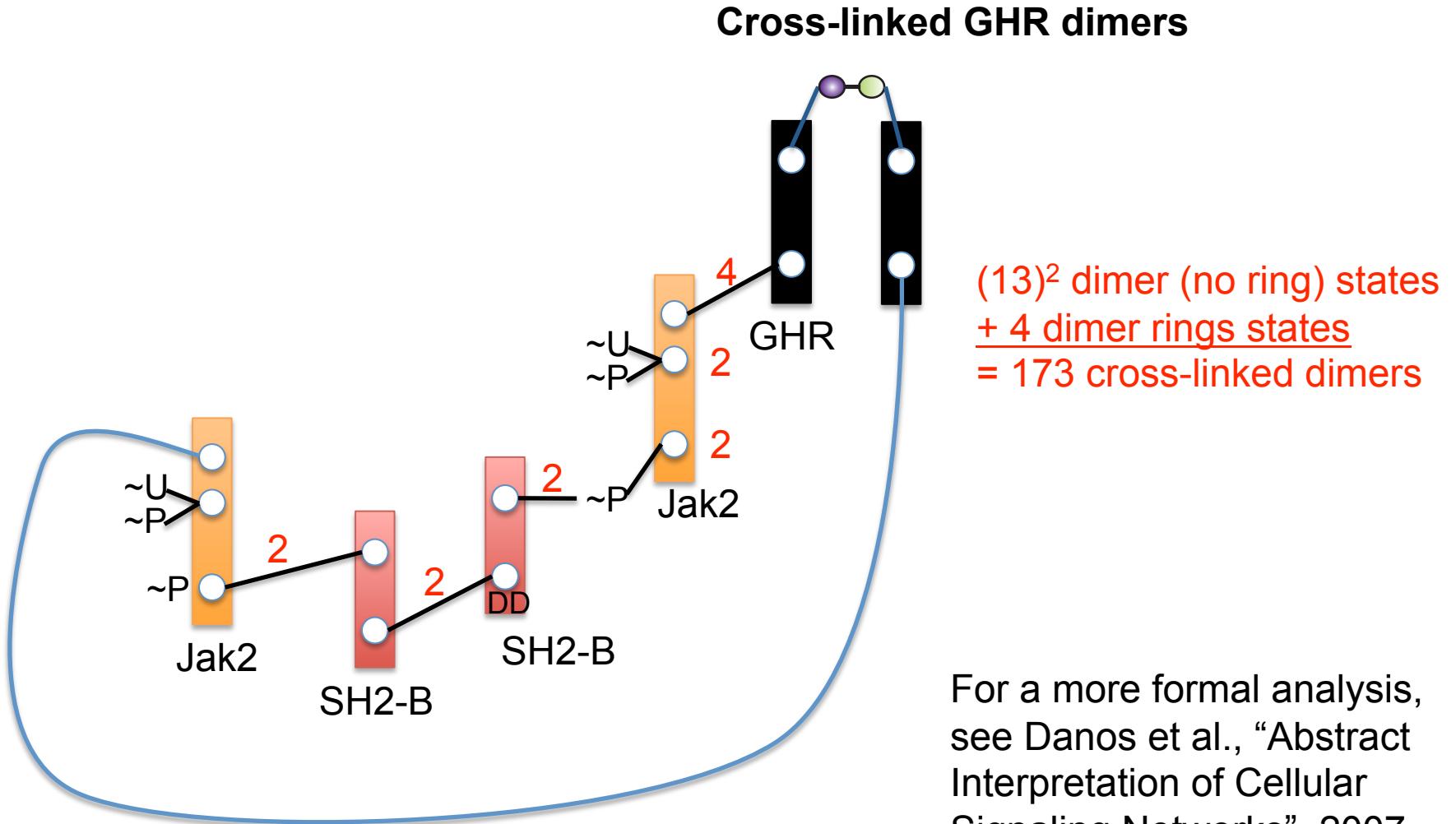
Combinatorial Complexity



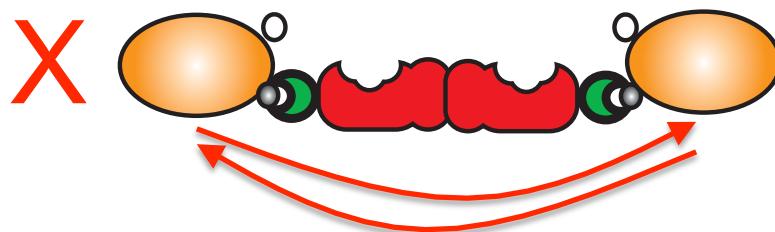
Combinatorial Complexity



Combinatorial Complexity

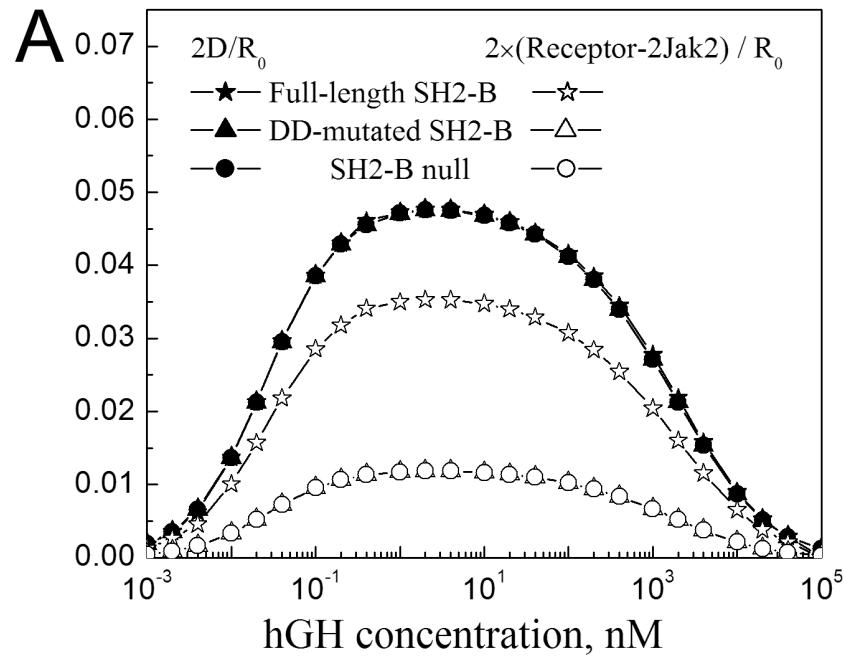


Model does not support the heterotetramer mechanism

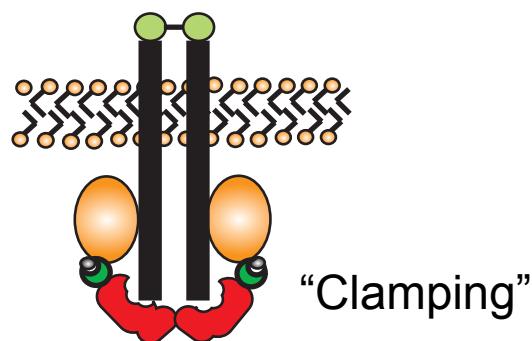
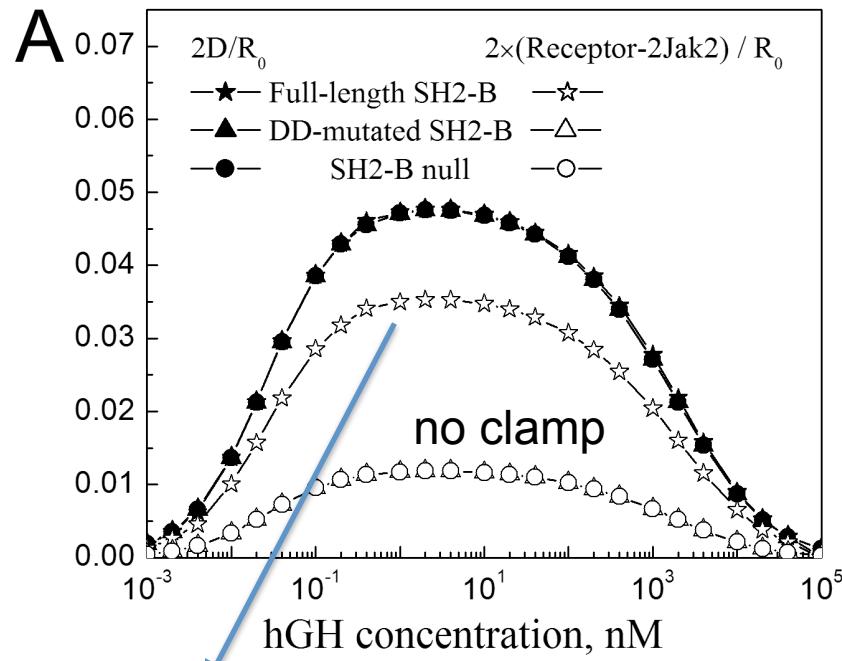


- Complex is short-lived and does not reach substantial concentration
- Observed phosphorylation is probably below detection limit, even when association rate increased
- Model may be an overestimate because it assumes Y813 is initially phosphorylated in the experiment

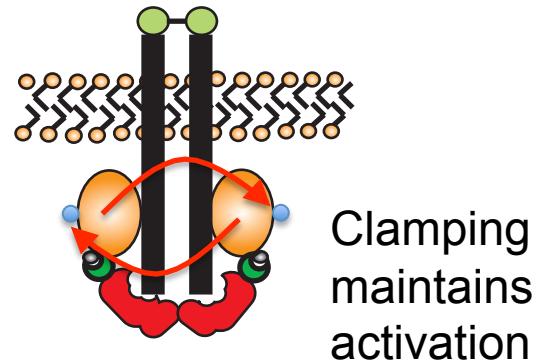
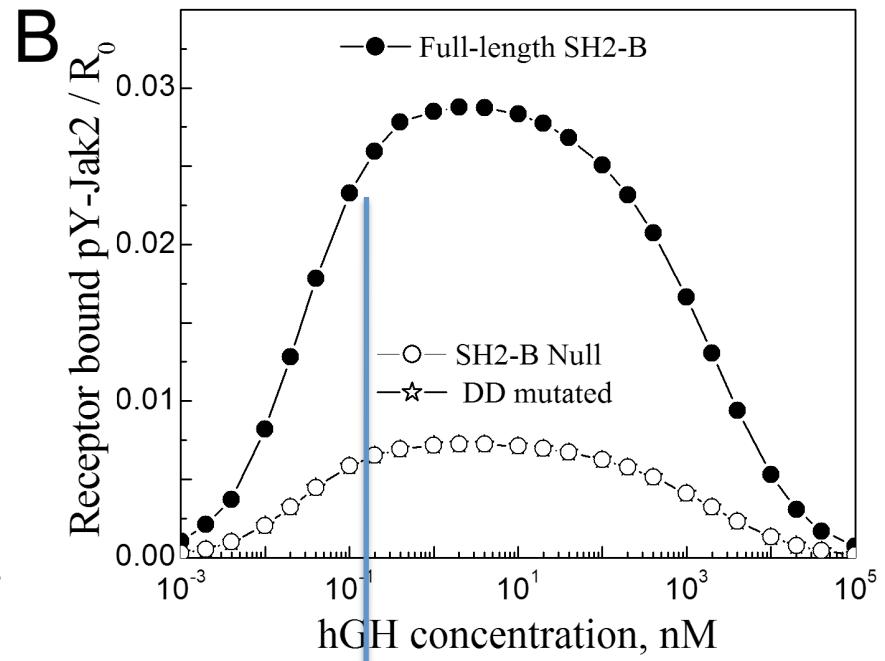
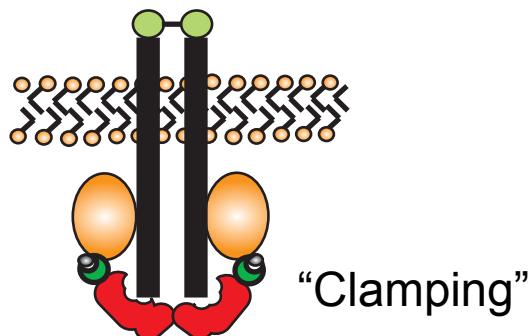
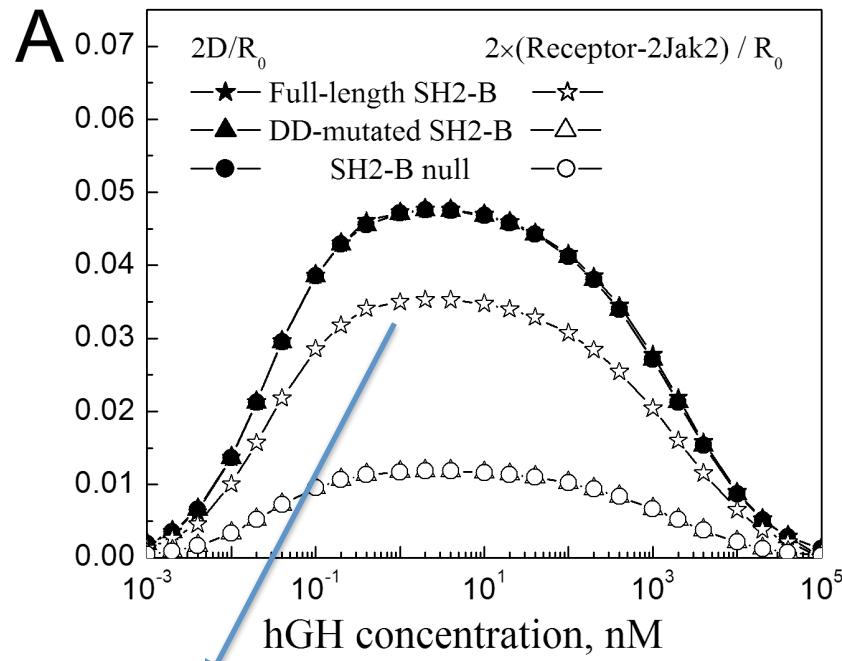
In Vivo Results: Basic Cellular Model



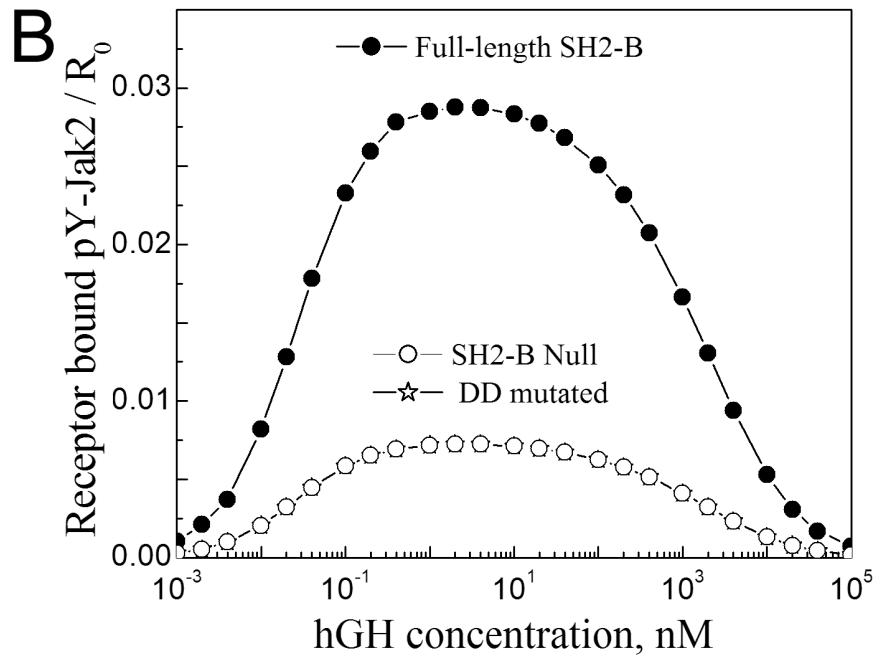
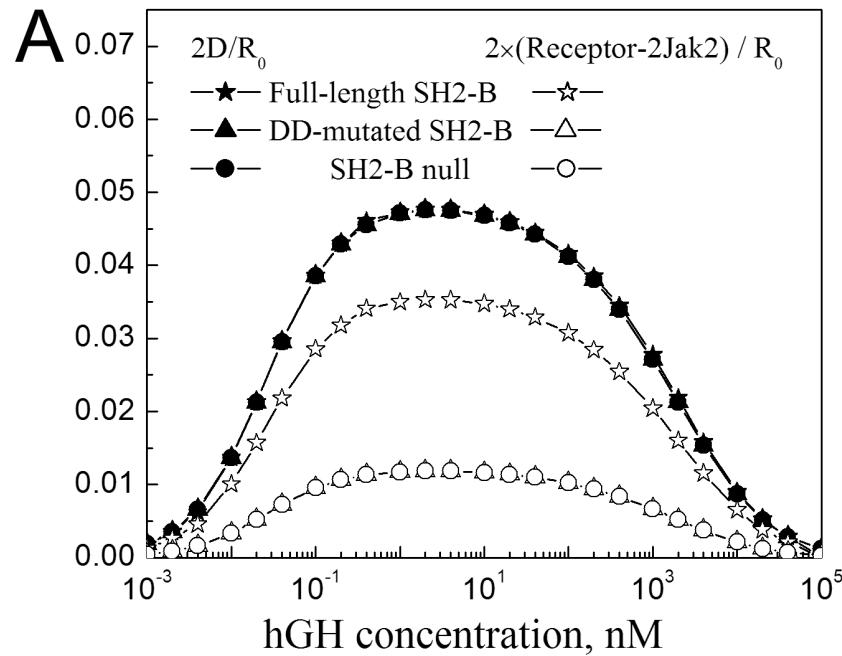
In Vivo Results: Basic Cellular Model



In Vivo Results: Basic Cellular Model

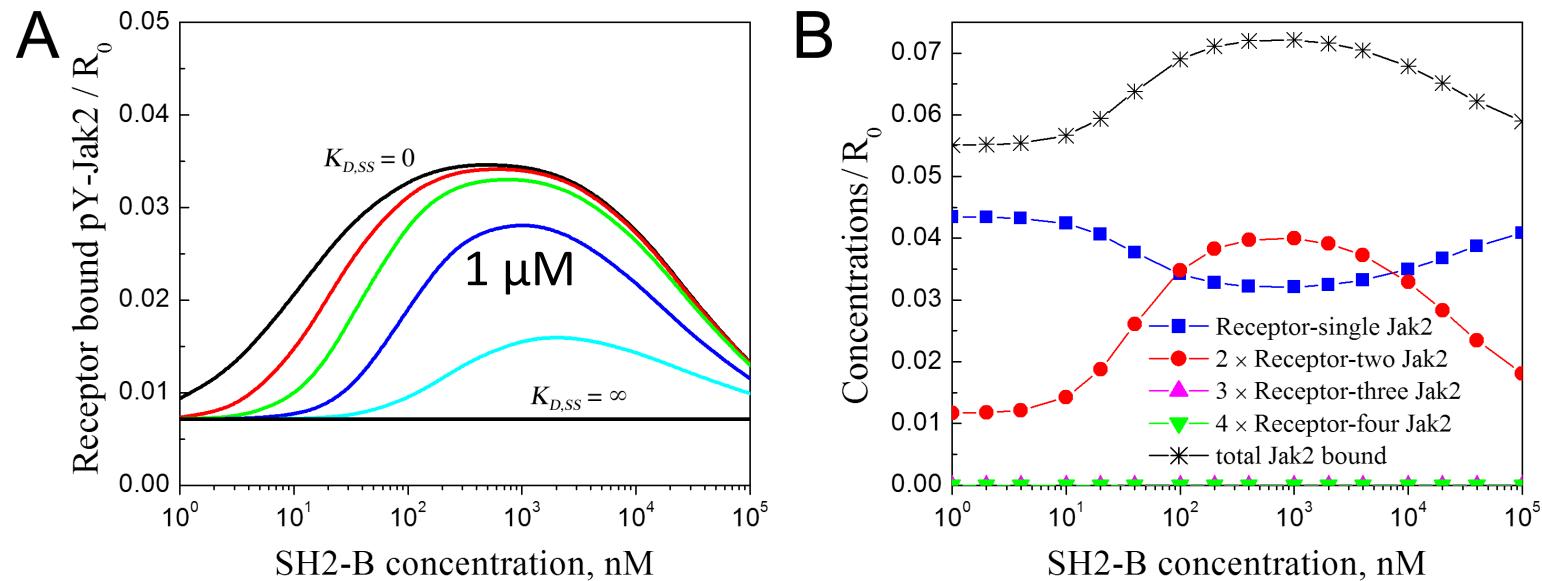


In Vivo Results: Basic Cellular Model

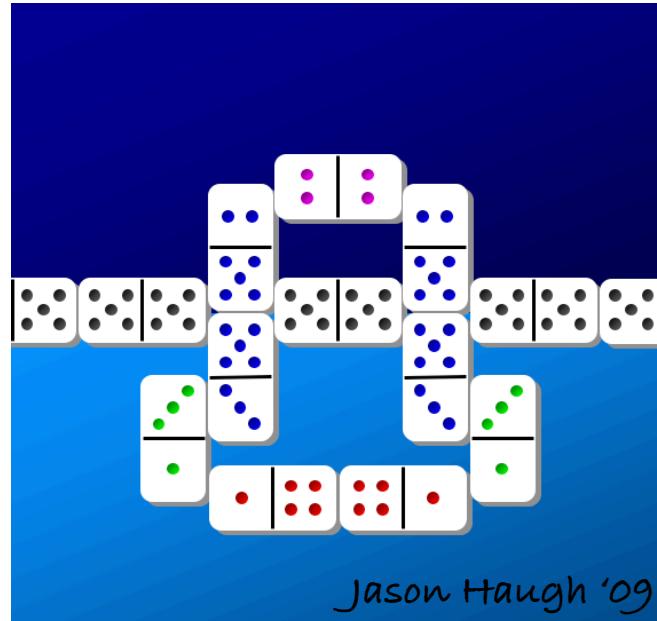


- Receptor dimerization is not affected
- Receptor-Jak2 tetramer is stabilized
- SH2-B gives about 4-fold increase in Jak2 activation
- SH2-B is effective only with intact dimerization domain

Analysis of Ring Formation

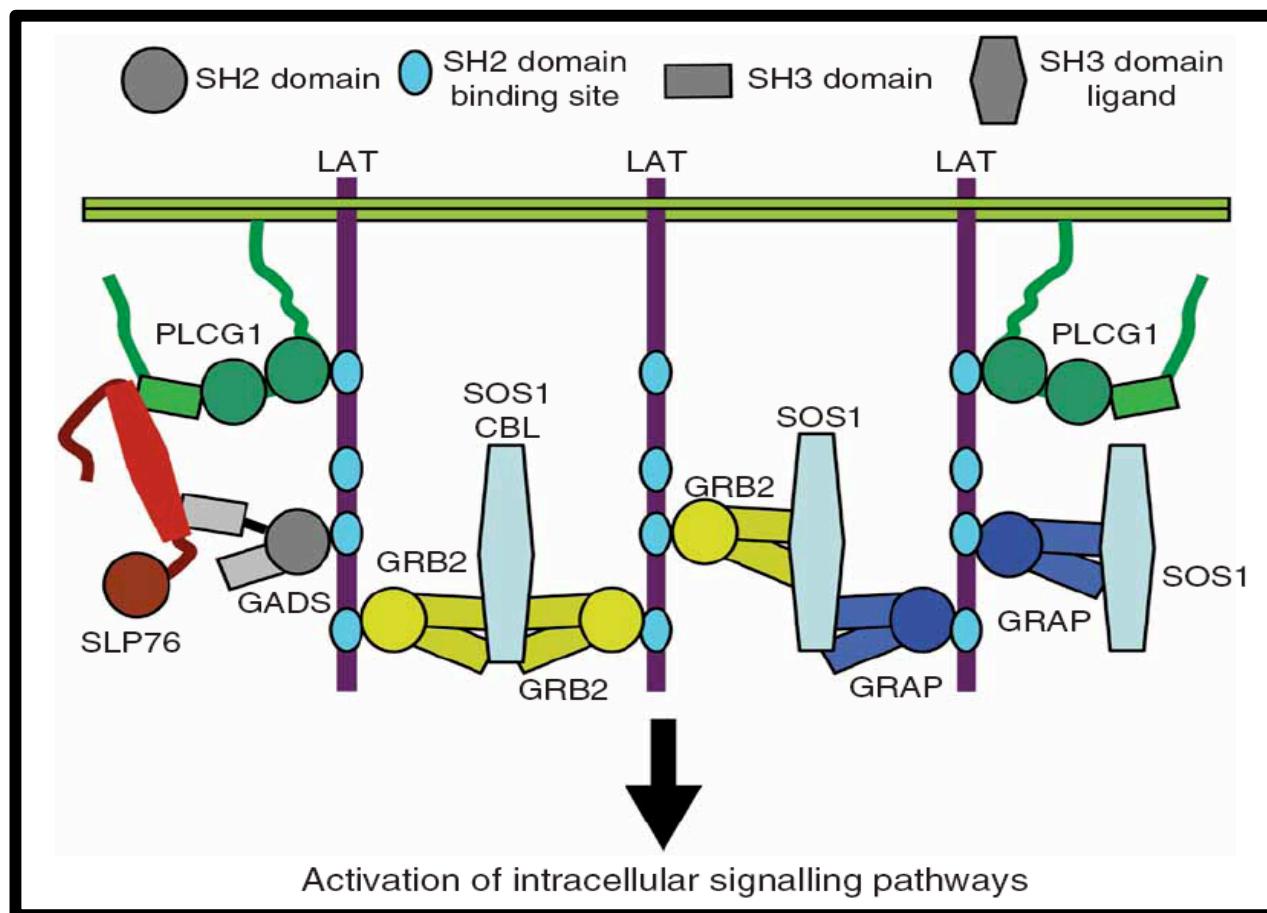


- *Affinity for dimerization can be weak*
- SH2-B only modestly affects overall Jak2 recruitment
- **Complexes with 2 Jak2 are preferentially stabilized**
- Recruitment of more than 2 Jak2 is rare
- cross-linking of receptor complexes is negligible



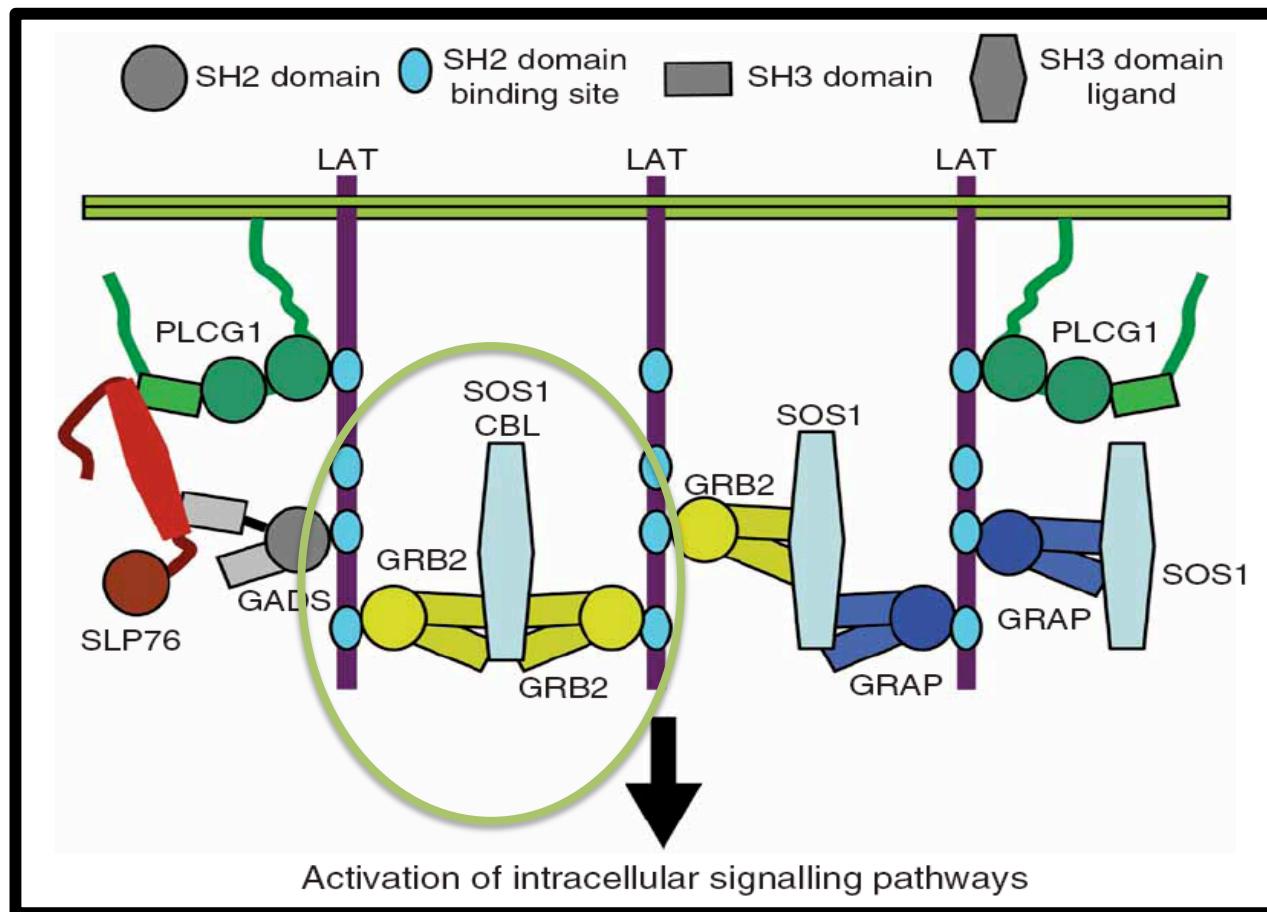
Jason Haugh '09

LAT oligomerization mediated by GRB2 and SOS1



Houtman et al., *Nat. Struct. Mol. Biol.*, 13, 798 (2006)

LAT oligomerization mediated by GRB2 and SOS1



Houtman et al., *Nat. Struct. Mol. Biol.*, 13, 798 (2006)

LAT oligomerization model

Formation of the 2:1 Grb2:Sos1 complex

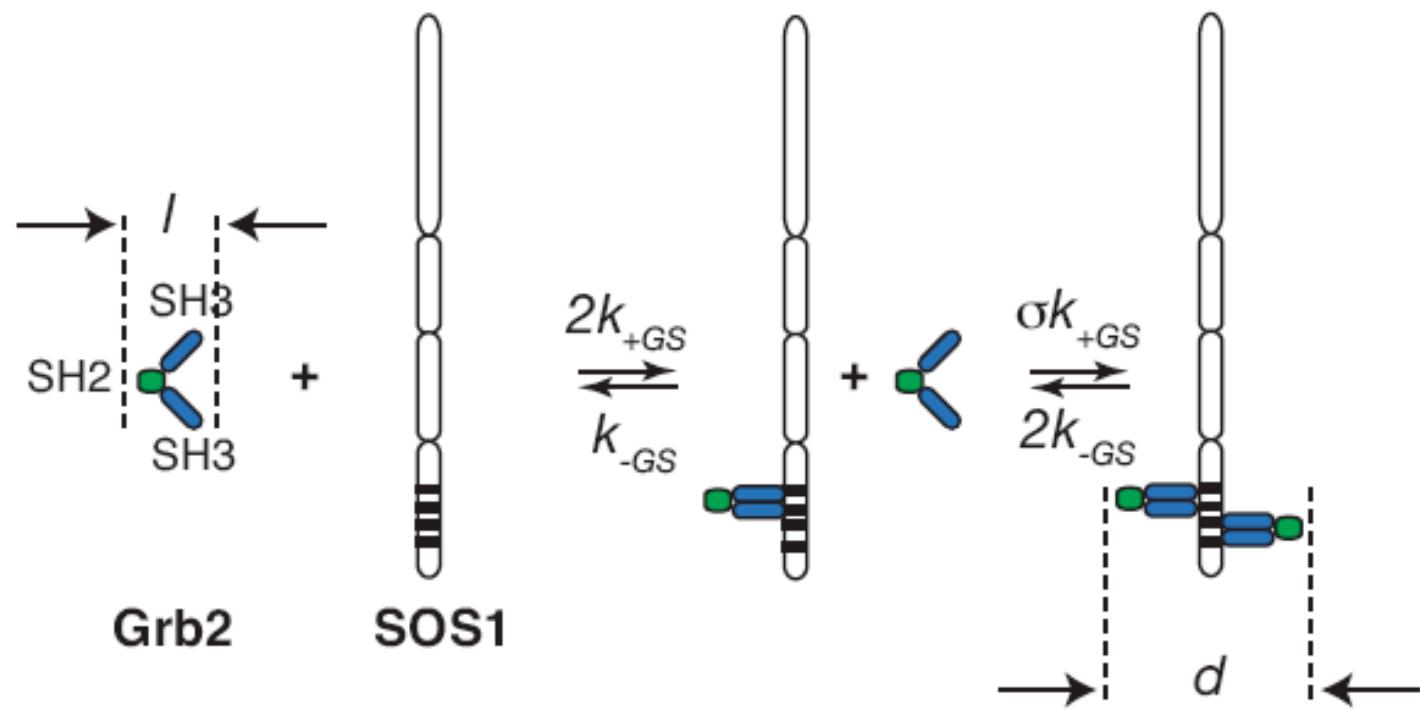
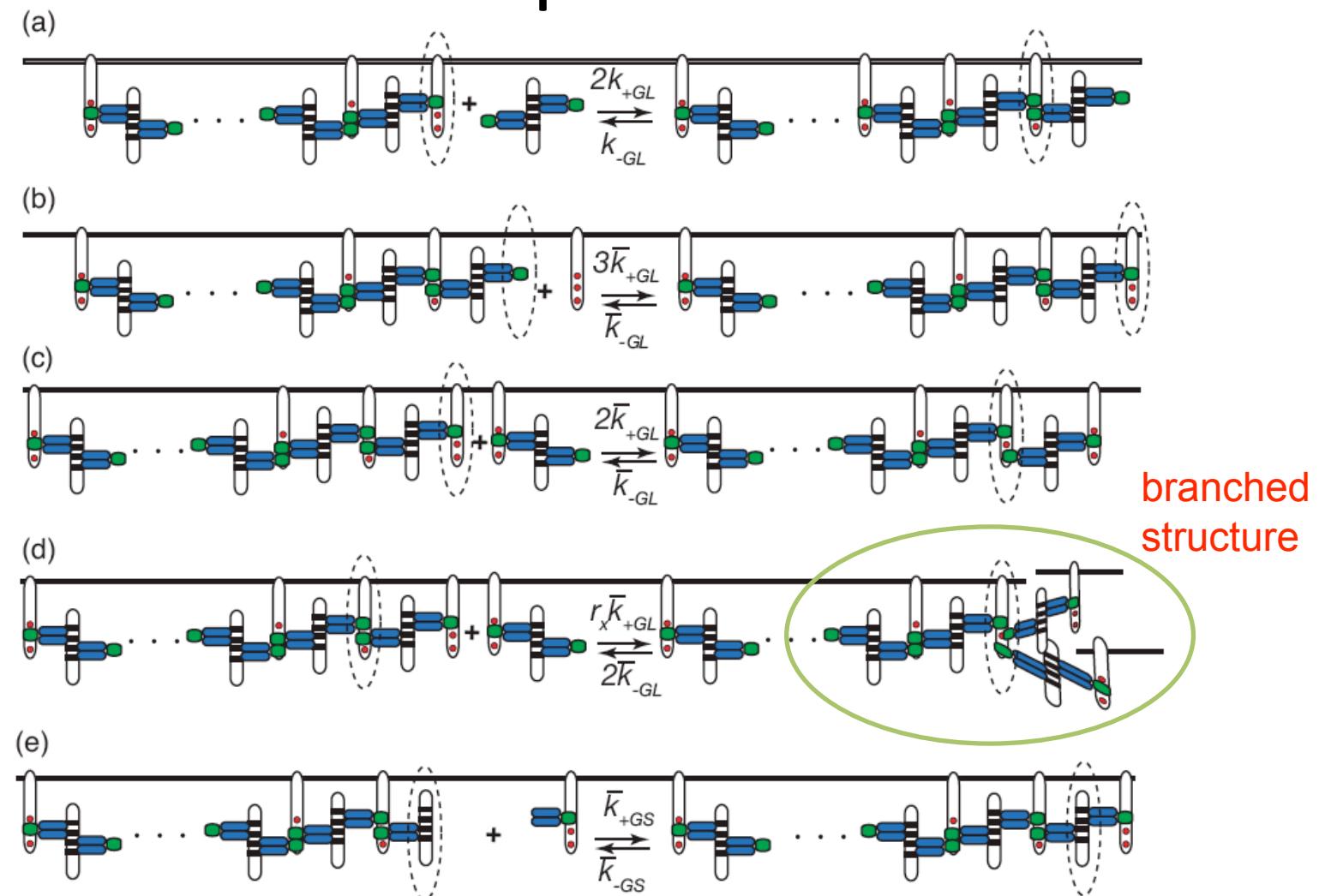


Figure 1.

Nag, Monine, Faeder, and Goldstein, *Biophys. J.* (2009)

Binding and cross-linking via the 2:1 complex



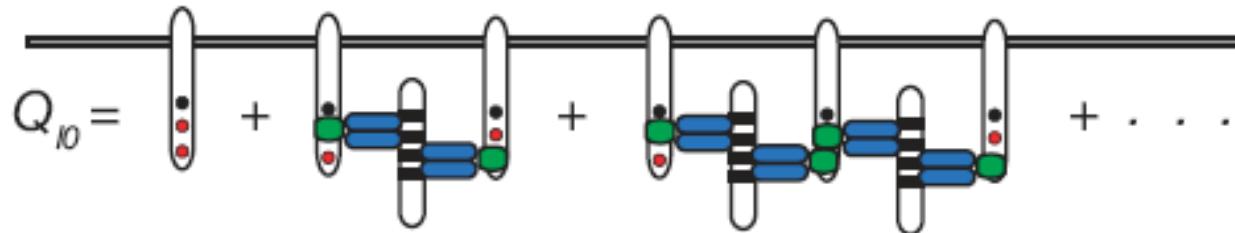
Nag, Monine, Faeder, and Goldstein, *Biophys. J.* (2009)

Parameters of the model

Parameter	Value	Reference
V	$3.6 \times 10^{-10} \text{ cm}^3$	(42)
A	$8.0 \times 10^{-6} \text{ cm}^2$	(42)
σ	0.5	(15)
K_{GL}	$3.4 \times 10^6 \text{ M}^{-1}$	(15, 21, 23)
k_{-GL}	0.31 s^{-1}	(43)
$k_{+GL} = k_{-GL} K_{GL}$	$1.1 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}$	
K_{GS}	$2.7 \times 10^6 \text{ M}^{-1}$	(15)
k_{+GS}	$9.5 \times 10^4 \text{ M}^{-1}$	(27)
$k_{-GS} = k_{+GS}/K_{GS}$	0.03 s^{-1}	
\bar{K}_{GL}	$1.7 \times 10^{15} \text{ mole}^{-1} \text{ cm}^2$	see caption ¹
\bar{k}_{-GL}	0.31 s^{-1}	
$\bar{k}_{+GL} = \bar{k}_{-GL} \bar{K}_{GL}$	$5.3 \times 10^{14} \text{ mole}^{-1} \text{ cm}^2 \text{ s}^{-1}$	

Equilibrium theory

Partition function = sum of aggregate concentrations



$$q = w + \frac{1}{\gamma q_0} \left[1 - \frac{1 - (1 - 4\gamma q_0^2)^{\frac{3}{2}}}{6\gamma q_0^2} \right]. \quad (24)$$

Conservation laws:

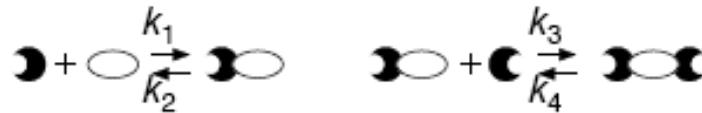
$$g + \mu g s + \sigma \mu \theta g^2 s + \left(\frac{A L_T}{V G_T} \right) g \left(\frac{\partial q_l}{\partial g} \right) = 1.$$

$$s + 2\theta g s + \sigma \theta^2 g^2 s + \left(\frac{A L_T}{V S_T} \right) s \left(\frac{\partial q_l}{\partial s} \right) = 1.$$

$$l \left(\frac{\partial q_l}{\partial l} \right) = w \left(\frac{\partial q_l}{\partial w} \right) = 1.$$

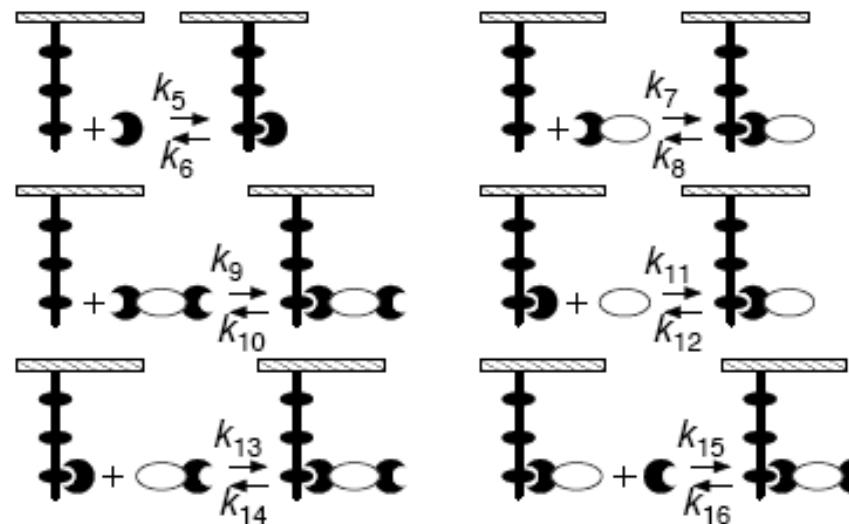
Rules for the kinetic model

In solution

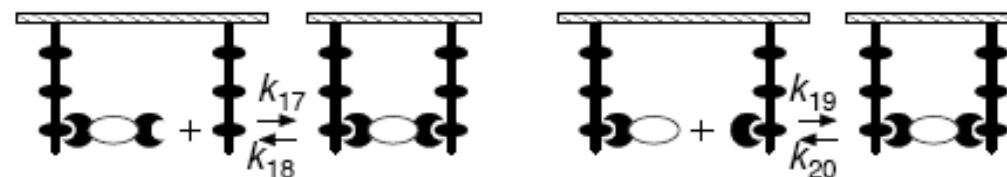


Notations:
LAT
SOS1
GRB2

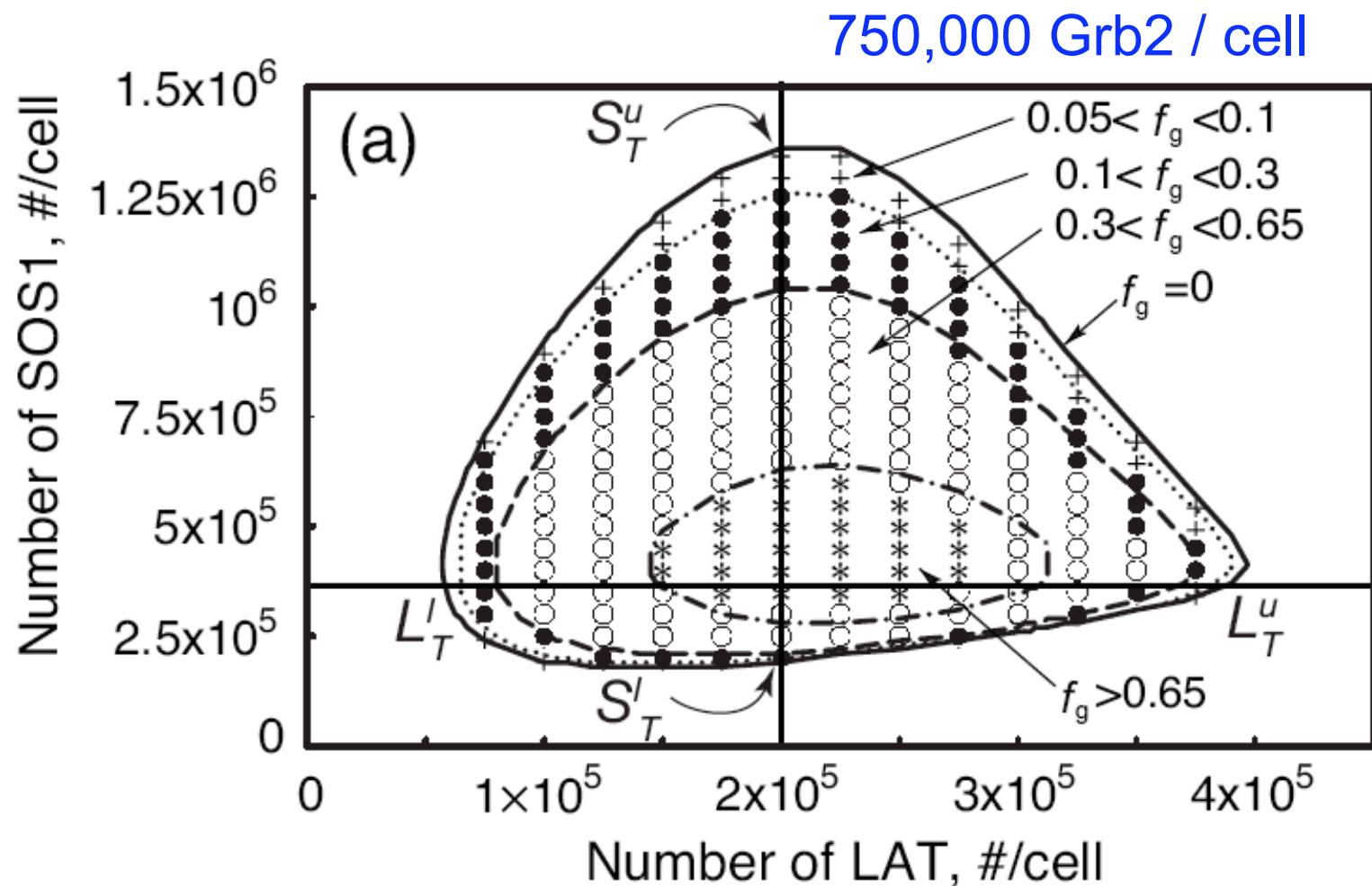
From solution to the membrane



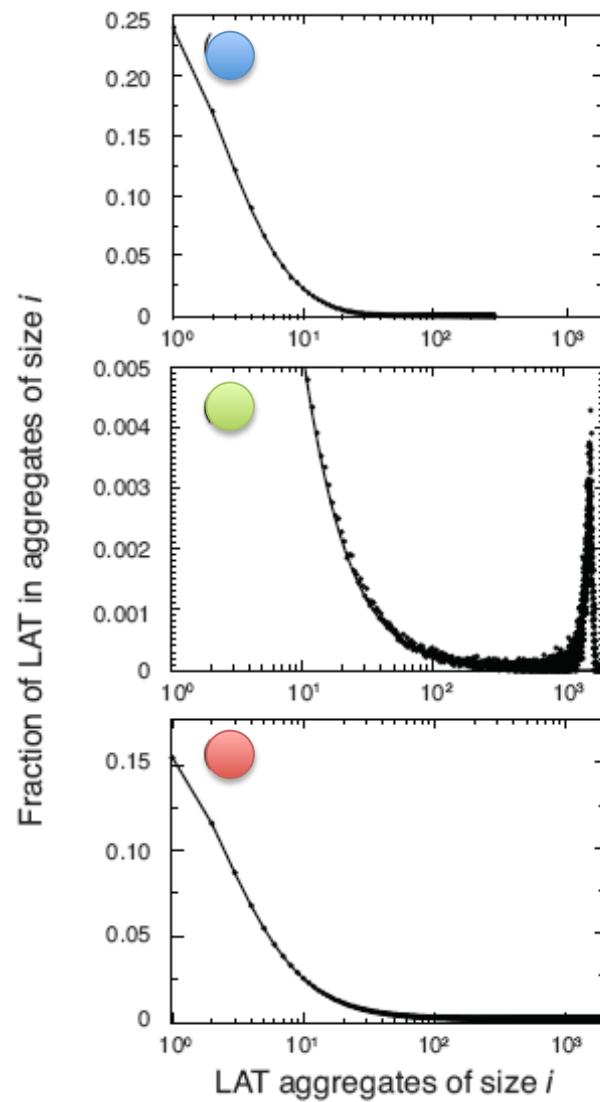
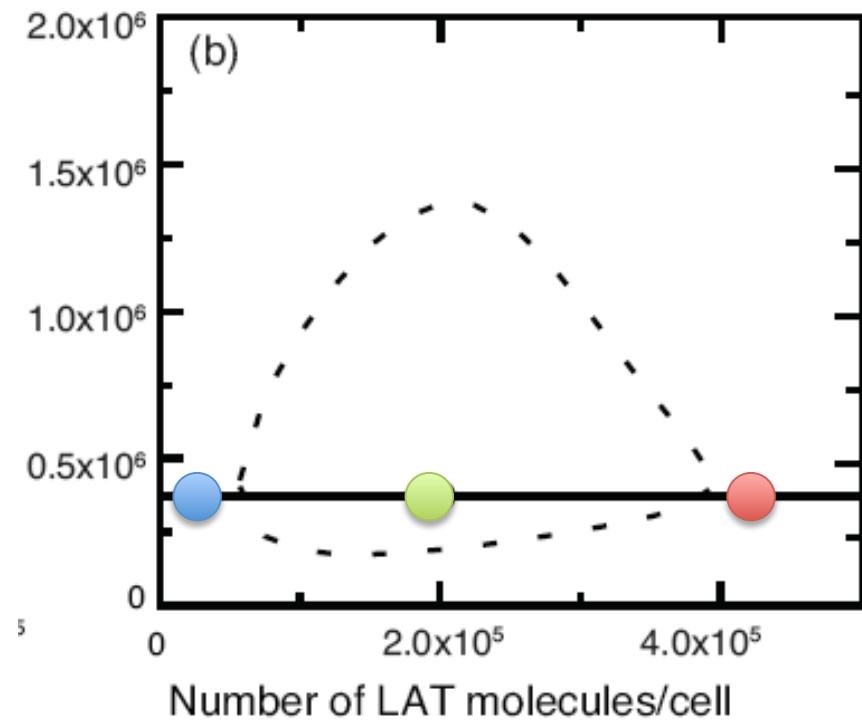
At the membrane



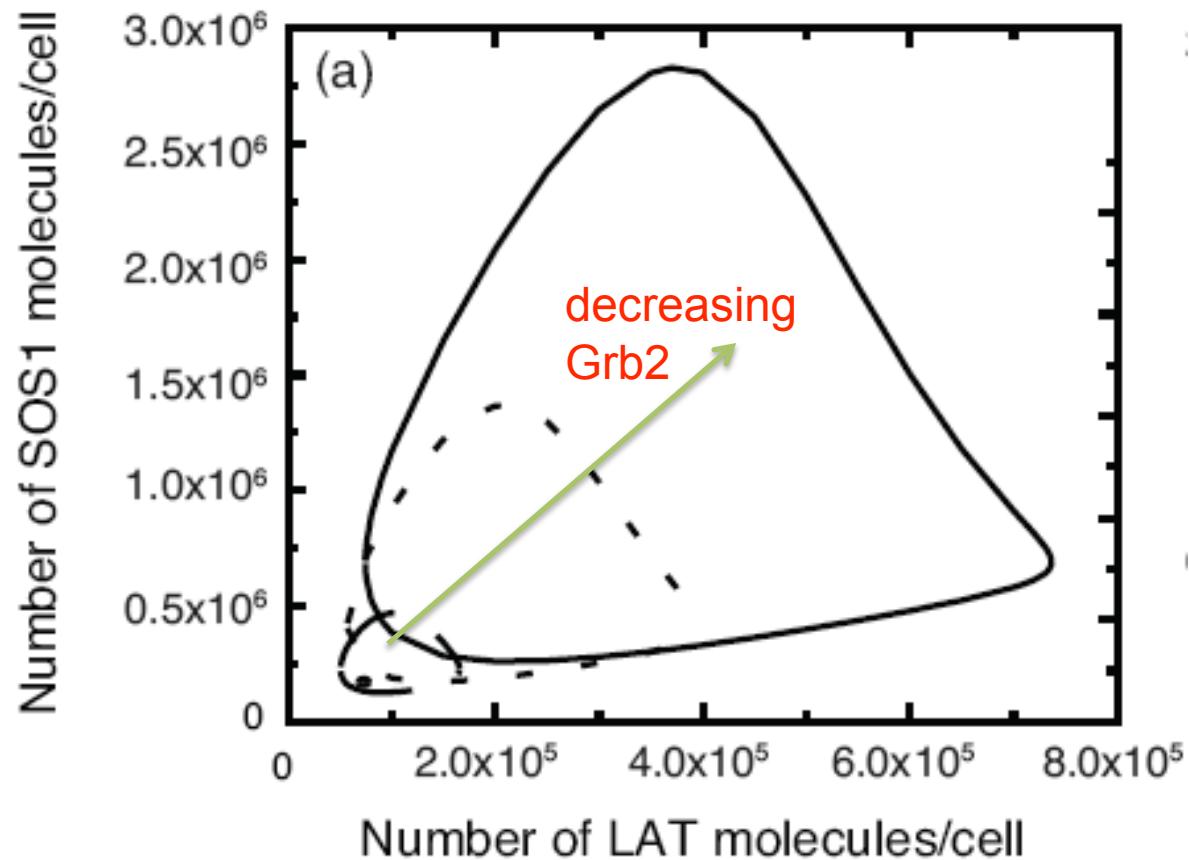
Phase diagram for sol-gel coexistence



Distribution of LAT aggregates



Area of gel phase is sensitive to Grb2 concentration



Do cells embrace or avoid this phenomenon?

Experimental observation of LAT aggregation

JCB
Article

High resolution mapping of mast cell membranes reveals primary and secondary domains of Fc ϵ RI and LAT

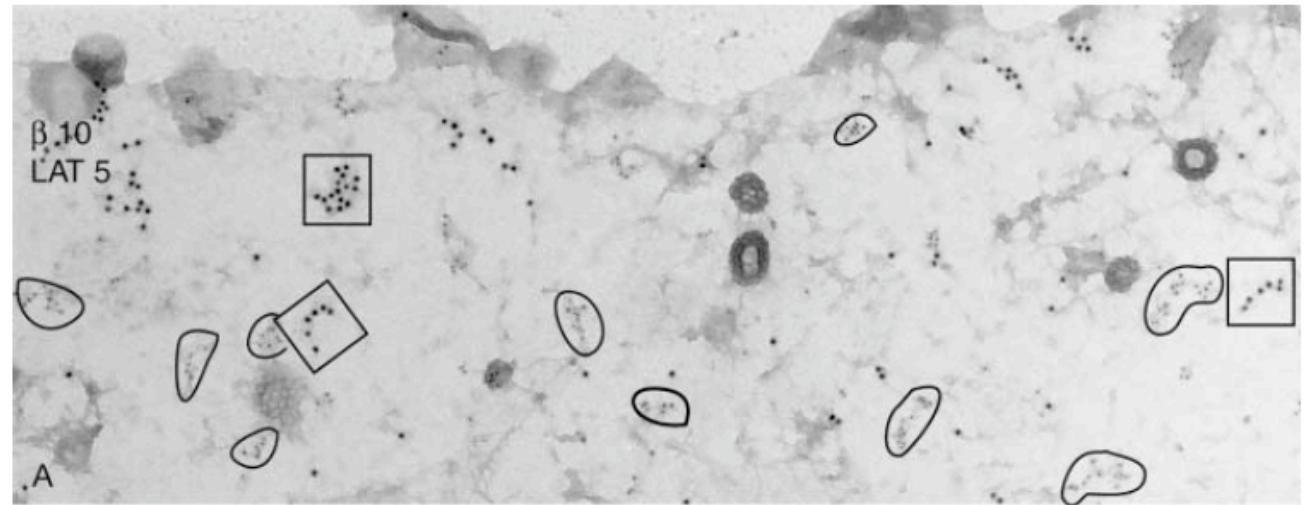
Bridget S. Wilson, Janet R. Pfeiffer, Zurab Surviladze, Elizabeth A. Gaudet, and Janet M. Oliver

Department of Pathology and Cancer Research and Treatment Center, University of New Mexico Health Sciences Center,
Albuquerque, NM 87131

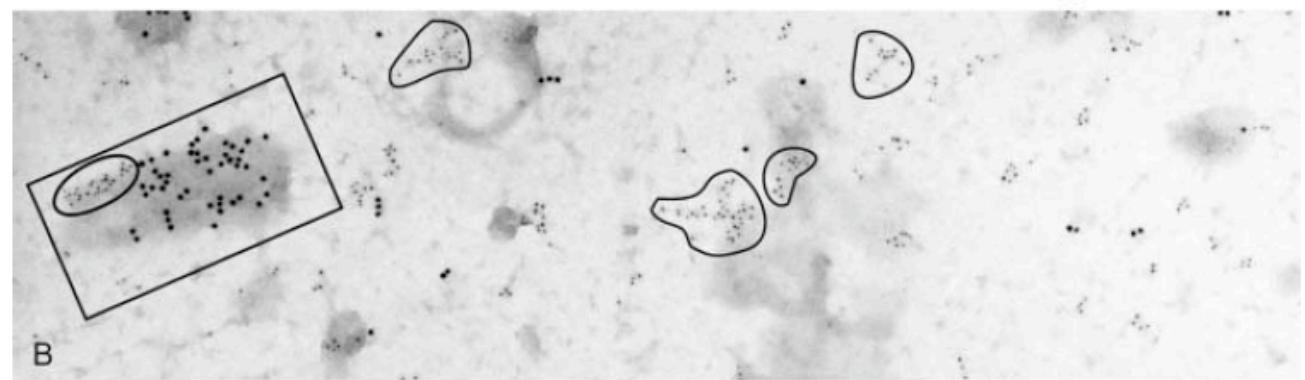
Wilson et al., *J. Cell Biol.* **154**, 645 (2001)

Experimental observation of LAT aggregation

Before Fc ϵ RI cross-linking

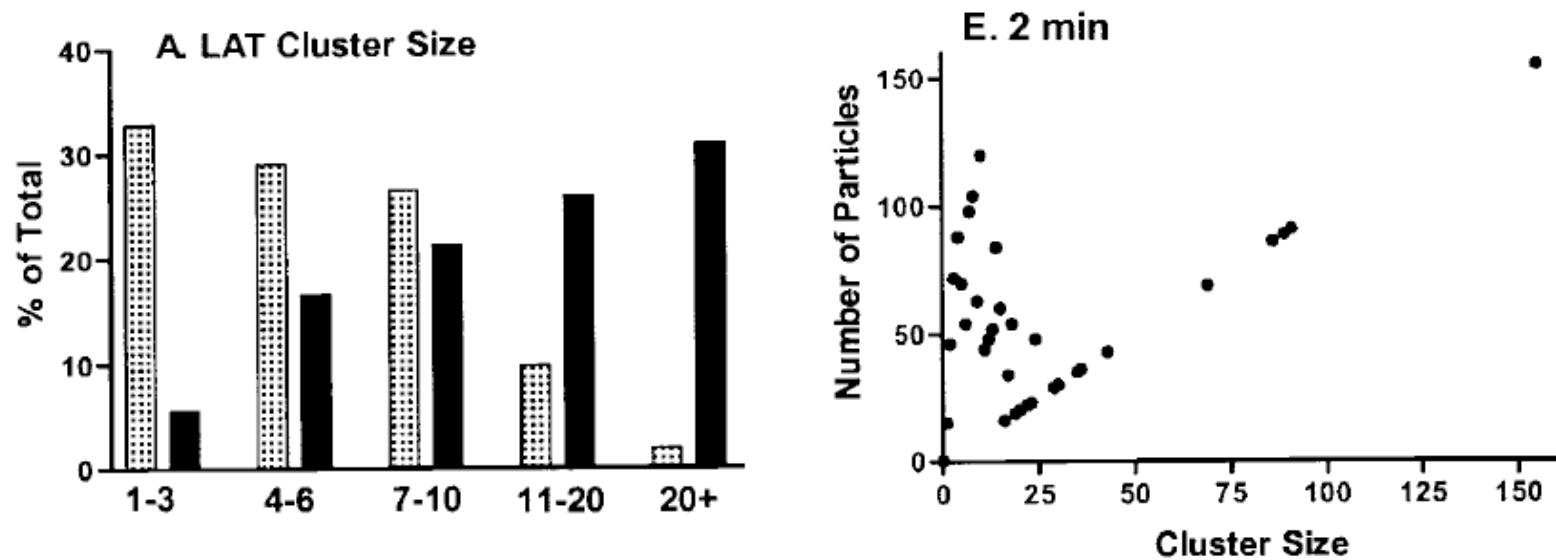


After Fc ϵ RI cross-linking



Wilson et al., *J. Cell Biol.* 154, 645 (2001)

Experimental observation of LAT aggregation

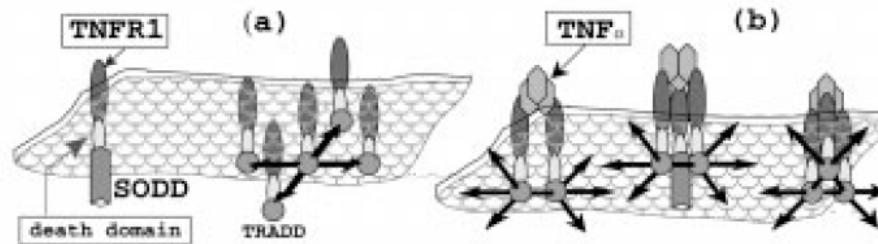


Wilson et al., *J. Cell Biol.* **154**, 645 (2001)

Open Problem

- Spatial simulations of LAT (and aggregation of other signaling proteins at the membrane)
- Possible approaches include
 - Brownian Dynamics simulations
 - Lattice-based (stat. mech.) simulations

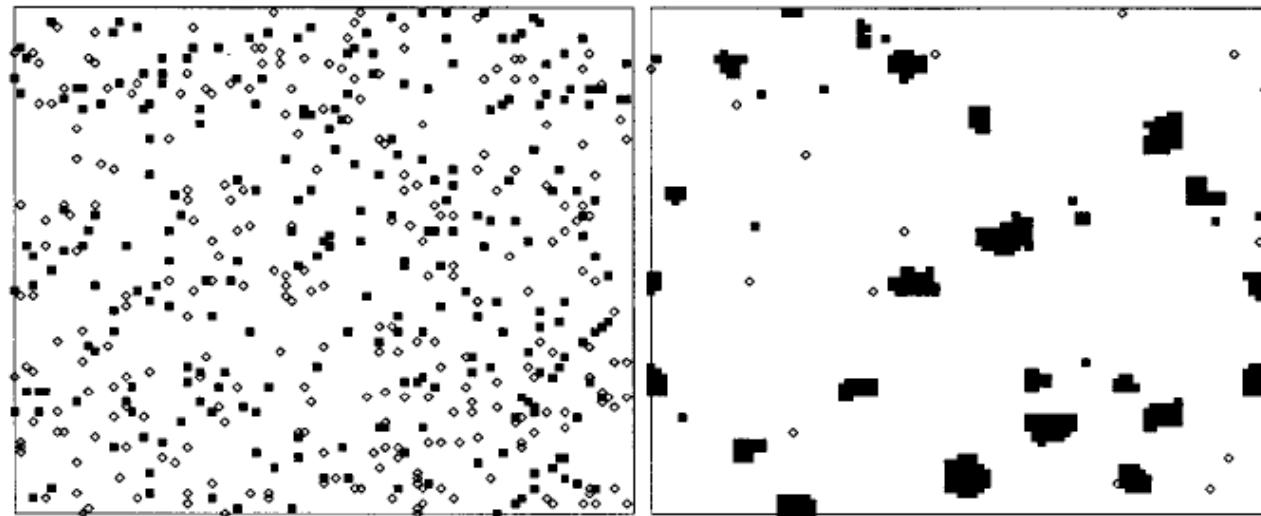
Statistical Mechanics Model for Receptor Clustering



$$H_0(\{n, t\}) = - \sum \mu(t_i) n_i \equiv - \sum \mu((R_k L_l S_m T_n)_i) n_i$$

B initial, $\langle n \rangle = 0.05$, $B = 1.0$

final, $\langle n \rangle = 0.05$, $B = 1.0$



Guo and Levine, *J. Biol. Phys.* **26**, 219-234 (2000)